

NAWAB SHAH ALAM KHAN COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF CIVIL ENGINEERING

(Name of the Subject/Lab Course): **STRENGTH OF MATERIAL - I**

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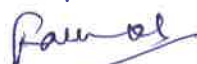
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2. SYLLABUS B.TECH

UNIT - I

SIMPLE STRESSES AND STRAINS: Elasticity and plasticity – Types of stresses and strains – Hooke's law – stress – strain diagram for mild steel – Working stress – Factor of safety – Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them – Bars of varying section – composite bars – Temperature stresses. Elastic constants.

STRAIN ENERGY: Resilience – Gradual, sudden, impact and shock loadings – simple applications.

UNIT - II

SHEAR FORCE AND BENDING MOMENT: Definition of beam – Types of beams – Concept of shear force and bending moment – S.F and B.M diagrams for cantilever, simply supported and overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads

– Point of contraflexure – Relation between S.F., B.M and rate of loading at a section of a beam.

UNIT - III

FLEXURAL STRESSES: Theory of simple bending – Assumptions – Derivation of bending equation: M/I

$= f/y = E/R$ - Neutral axis – Determination of bending stresses – Section modulus of rectangular and circular sections (Solid and Hollow), I, T, Angle and Channel sections – Design of simple beam sections.

SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.

UNIT - IV

PRINCIPAL STRESSES AND STRAINS: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses – Normal and tangential stresses on an inclined plane for biaxial stresses

– Two perpendicular normal stresses accompanied by a state of simple shear – Mohr's circle of stresses
– Principal stresses and strains – Analytical and graphical solutions.

THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory, Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory (Von Mises Theory).

UNIT – V

DEFLECTION OF BEAMS : Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam – Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load-Mohr's theorems – Moment area method – application to simple cases including overhanging beams.

CONJUGATE BEAM METHOD: Introduction – Concept of conjugate beam method, Difference between a real beam and a conjugate beam, Deflections of determinate beams with constant and different moments of inertia.

Textbooks:

1. Mechanics of Materials by B. C. Punmia, A. K. Jain and A. K. Jain, Laxmi Publications (P) Ltd, New Delhi.
2. Strength of Materials – A Practical Approach Vol.1 by D. S. Prakash Rao, Universities Press, Hyderabad, India.
3. Mechanics of materials by F. Beer, E. R. Johnston and J. Dewolf (Indian Edition – SI units), Tata McGraw Hill Publishing Co. Ltd., New Delhi

Reference Books:

1. Mechanics of Materials (SI edition) by J.M. Gere and S. Timoshenko, CL Engineering, India
2. Engineering Mechanics of Solids by E. G. Popov, Pearson Education, New Delhi, India
3. Strength of Materials by R. K. Bansal, Laxmi Publications (P) Ltd., New Delhi, India.
4. Strength of Materials by S.S.Bhavikatti, Vikas Publishing House Pvt. Ltd.
5. Strength of Materials by R.K Rajput, S.Chand & Company Ltd.
6. Strength of Materials by R. S. Khurmi, S. Chand publication New Delhi, India.
7. Fundamentals of Solid Mechanics by M.L.Gambhir, PHI Learning Pvt. Ltd
8. Strength of Materials by R.Subramanian, Oxford University Press.

2. SYLLABUS B.E

UNIT-I

Simple Stresses and Strains: Definitions of stresses and strains, Hooke's Law, Modulus of Elasticity, Stress

- Strain curve for ductile materials, Elastic constants, compound bars and temperature stresses.

Strain Energy: Strain energy and resilience in statically determinate bars subjected to gradually applied, suddenly applied, impact and shock loads.

UNIT-II

Compound Stresses: Stresses on oblique planes, principal stresses and planes. Mohr's circle of stress. Theories of Failure based on maximum principal stress, maximum principal strain, maximum shear stress, maximum strain energy and maximum shear strain energy

Application to pressure vessels: Thin cylinders subjected to internal fluid pressure, volumetric change. Thick Cylinders: Lamé's equations, stresses under internal and external fluid pressures, Compound cylinders, Shrink fit pressure.

UNIT-III

Shear Force and Bending Moment: Different types of beams and loads, shear force and bending moment diagrams for cantilever, and simply supported beams with and without overhangs subjected to different kinds of loads viz., point loads, uniformly distributed loads, uniformly varying loads and couples.

Bending Stresses in Beams: Assumptions in theory of simple bending, Derivation of flexure equation, Moment of resistance, calculation of stresses in statically determinate beams for different loads and different types of structural sections.

UNIT-IV

Shear Stress in Beams: Derivation of equation of shear stresses, distribution across rectangular, circular, T and I section.

Direct and Bending Stresses: Direct loading, Eccentric loading, limit of eccentricity, Core of sections, rectangular and circular, solid and hollow sections

UNIT-V

Torsion: Theory of pure torsion in solid and hollow circular shafts, shear stress, angle of twist, strength and stiffness of shafts, Transmission of Power. Combined torsion and bending with and without end thrust. Determination of principal stresses and maximum shear stress. Equivalent bending moment and equivalent twisting moment.

Springs: Close and open coiled helical springs under axial load and axial twist, Carriage springs.

Suggested Readings:

1. D.S. Prakash Rao, *Strength of Materials- A Practical Approach*, Universities Press, 1999.

2. R.K. Rajput, *A Textbook of Strength of Materials*, S. Chand Publications, 2007.
3. R. Subramanian, *Strength of Materials*, Oxford University Press, New Delhi 2005.
4. S. S. Bhavikatti, *Strength of materials*, Vikas Publishing House, 2002.
5. Ferdinand P Beer, Johnston and De Wolf., *Mechanics of Materials*, Tata McGraw-Hill, 2004.

3. INSTITUTIONAL VISION AND MISSION

Vision:

To impart quality technical education with strong ethics, producing technically sound engineers capable of serving the society and the nation in a responsible manner.

Mission of Institute

- M1:** To provide adequate knowledge encompassing sound technical concepts and soft skills thereby inculcating sound ethics.
- M2:** To provide a conducive environment to nurture creativity in teaching- learning process.
- M3:** To identify and provide opportunities for deserving students of all communities.
- M4:** To strive and contribute to the needs of the society and the nation by applying advanced engineering and technical concepts.

4. DEPARTMENT VISION AND MISSION

VISION:

To develop technically strong civil engineers having ethics and human values by providing quality education, enabling them to be self strong in facing any challenges that may arise during their service in particular to the society and in general to the nation.

MISSION:

- M1:** To provide conceptionally strong technical knowledge relating to all fields of civil engineering braced with professional ethics.
- M2:** To adopt the latest developments in civil engineering to provide conducive environment for better teaching learning process.
- M3:** To provide adequate soft skills and make the students industry ready to grab the opportunities in this field.
- M4:** To encourage students to participate in various technical events at research institutes, institutes of higher learning so that they develop the capabilities to serve the nation effectively.

5. Program Outcomes (C. E. D)

1. **ENGINEERING KNOWLEDGE:** To apply the knowledge of various fields such as Science, Mathematics & Engineering, and to provide specific solutions for engineering problems.
2. **PROBLEM ANALYSIS:** An ability to design a system and to solve basic engineering problems, which are required for all Engineering fields.
3. **DESIGN & DEVELOPMENT OF SOLUTIONS:** To provide solutions for engineering problems the design basic system and components to meet the requirements of public safety & environmental concerns.
4. **CONDUCT INVESTIGATIONS OF COMPLEX PROBLEMS:** To apply the basic knowledge of engineering to design & conduct experiments to solve the complex engineering problems.
5. **MODERN TOOL USAGE:** To apply the modern engineering tools in the field of engineering to solve basic and complex engineering problems.
6. **THE ENGINEER & SOCIETY:** To understand the responsibility related to professional engineering practices.
7. **ENVIRONMENT & SUSTAINABILITY:** To provide the impact of engineering solutions in global & environmental contexts.
8. **ETHICS:** To know the social responsibilities and apply ethical principles and commitment in practice.
9. **INDIVIDUAL & TEAM WORK:** To function effectively as an individual and also as a member of a diverse team.
10. **COMMUNICATION:** To communicate effectively in various complex engineering activities and give an innovative idea for any presentation or documentation activities.
11. **PROJECT MANAGEMENT & FINANCE:** An ability to initiate the interdisciplinary projects in various engineering fields and demonstrate knowledge and understanding of the engineering and management principles.
12. **LIFE LONG LEARNING:** To have preparation and ability to engage in independent and life-long learning in context of technological revolution.

Program Specific Outcomes (PSO):

PSO1: To plan the building and perform analysis, design, estimation and execute all kinds of civil Engineering Projects.

PSO2: To adopt new innovative technology and use modern techniques, so as to execute the project within the stipulated time.

6.Course Objectives

At the end of the course, the student will be able to:

- CO1: Relate mechanical properties of a material with its behaviour under various load types and classify the types of material according to the modes of failure and stress-strain curves.
- CO2: Apply the concepts of mechanics to find the stresses at a point in a material of a structural member and Create diagrams for shear force, bending moment, stress distribution, mohr's circle, elastic curve.
- CO3: Analyze a loaded structural member for deflections and failure strength.
- CO4: Evaluate the stresses & strains in materials and deflections in beam members.

Course Outcomes

- CO1: Students will be able **Calculate** the stress and strain developed in any structural member due to applied external load and can Differentiate the type of beams, and the various loading and support condition upon them
- CO2: Students will be able to **Apply** the formulae for beams under different loading condition. Draw shear force diagram and bending moment diagram for different type of beams.
- CO3: Students will be able **Derive** the pure bending equation, and on its basis explain the existence of normal stresses and shear stresses in the different layers of the beam and Explain the importance of and evaluate the section modulus for various beam cross-sections.

CO4: Students will be able to **Calculate** the normal and tangential stresses on an inclined section of a bar under uniaxial, biaxial, pure shear and plain stress conditions and can Evaluate the principal stress and principal strain at a point of a stressed member and draw the Mohr's circle of stresses.

7. CO – PO Mapping

CO-PO MAPPING

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	2	1	-	2	1	-	1	-	1	2
CO2	3	1	2	1	-	1	-	2	2	1	1	1
CO3	3	2	1	2	-	1	-	1	1	2	2	2
CO4	3	2	1	2	-	2	1	2	1	2	1	2
AVG	3	1.5	0.25	1.25	-	1.5	0.5	1.25	1.25	1.25	1.5	2.25

CO-PSO MAPPING

	PSO1	PSO2
CO1	3	2
CO2	3	2
CO3	2	2
CO4	2	1
AVG	2.5	1.75

Faculty Sign

IQAC Member Sign

H.O.D

Principal

8. Brief notes on the Importance of the Course

Civil Engineers are required to learn the fundamentals of design, analysis, and proportioning of reinforced concrete members and structures. Learn design concepts and modes of failure. Methods for analysis and design of these elements under flexure, shear, and axial loads will be examined. Learn how to make design decisions considering realistic constraints such as safety, economy and serviceability. Learn how to use the latest technology in solving structural analysis and design problems. To impart adequate knowledge on how to analyze and design reinforced concrete members and connection.

To understand the mechanical properties of structural concrete. To understand the behaviour of reinforced concrete elements under normal force, shear, moment and torsion. Concept of ultimate design of reinforced concrete beams, floor systems and columns are to be understood. To develop an understanding of and appreciation for basic concepts in the behavior and design of reinforced concrete systems and elements. To help the student develop an intuitive feeling about structural and material wise behavior and design of reinforced concrete systems and elements.

9. Prerequisites :

Engineering Mechanics

10. Class Timetables

10.1 Class Timetables 18-19

Branch	Civil II-A	DEPARTMENT OF CIVIL ENGINEERING						
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:30	1:30 TO 2:20	2:20 TO 3:10	3:10 TO 4:00
MON	BMCP		SM		LUNCH BREAK	M-4		SUR
TUE	SUR	GS LAB/CAD LAB				M-4		SM
WED	BMCP		SM			GS LAB/CAD LAB		
THUR	FM	SUR/SM LAB				FM		M-4
FRI	FM	SUR/SM LAB				SUR		FM
SAT	SM-1			FM-1				

Branch	Civil II-B	DEPARTMENT OF CIVIL ENGINEERING							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:30	1:30 TO 2:20	2:20 TO 3:10	3:10 TO 4:00	
MON	M-4		BMCP		LUNCH BREAK	SM			
TUE	FM		BMCP			SUR/SM LAB			
WED	M-4	SUR/SM LAB				M-4		FM	
THUR	SUR	CAD/GS LAB				SM		SUR	
FRI	SUR		FM			CAD/GS LAB			
SAT	SM-1					FM-1			

Branch	Civil II- C	DEPARTMENT OF CIVIL ENGINEERING						
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:30	1:30 TO 2:20	2:20 TO 3:10	3:10 TO 4:00
MON	FM		SUR		LUNCH BREAK	SUR/SM LAB		
TUE	SM	SUR/SM LAB				M-4		FM
WED	FM		BMCP			SM		M-4
THUR	BMCP		M-4			CAD/GS LAB		
FRI	SUR		SM			CAD/GS LAB		
SAT	SM-1					FM-1		

10.2 Class Timetables 19-20

DEPARTMENT OF CIVIL ENGINEERING							wef:15-7-2019		
Branch	Civil II-A					G-BLOCK (1st flr) , LH-13			
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10	
MON	SUR		FM		BREAK LUNCH	SUR/EG LAB			
TUE	SUR		SM-I			SM/EG LAB			
WED	EG		P & S			SUR/SM LAB			
THUR	SM-I		P & S			C.I			
FRI	FM		EG			LIBRARY		SPORT	

DEPARTMENT OF CIVIL ENGINEERING							wef:15-7-2019		
Branch	Civil II-B					G-BLOCK (1st flr) , LH-14			
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10	
MON	FM		P & S		BREAK LUNCH	SM		P & S	
TUE	EG	SUR/EG LAB				C.I			
WED	FM		SM			SUR		P & S	
THUR	SUR		EG			SUR/SM LAB			
FRI	EG	SM/EG LAB				LIBRARY		SPORT	

DEPARTMENT OF CIVIL ENGINEERING							wef:15-7-2019		
Branch	Civil II-C					G-BLOCK (1st flr) , LH-14			
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10	
MON	FM		P & S		BREAK LUNCH	SM/EG LAB			
TUE	EG	SUR/SM LAB				SUR/EG LAB			
WED	FM		SM			SUR		P & S	
THUR	SUR		EG			SM		P & S	
FRI	EG	C.I				LIBRARY		SPORT	

10.3 Class Timetables 20-21

TIME TABLE FOR B.E II YEAR I SEMESTER 2020 - 2021 (ONLINE MODE)								
DEPARTMENT OF CIVIL ENGINEERING								
Branch	Civil II-A							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON	IP		EG		LUNCH BREAK	EM	SUR & GEO	
TUE	SUR & GEO		SM			EM	EG	
WED	SM		OCE			BE	ES & E	
THUR	OCE		ES & E			BE	BE	
FRI	SM	SUR & GEO	IP			TUTORIALS		

TIME TABLE FOR B.E II YEAR I SEMESTER 2020 - 2021 (ONLINE MODE)								
DEPARTMENT OF CIVIL ENGINEERING								
Branch	Civil II-B							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON	SUR & GEO		EG		LUNCH BREAK	OCE	SM	
TUE	SM		EG			OCE	IP	
WED	EM		OCE			SUR & GEO	BE	
THUR	EM		IP			SUR & GEO	ES & E	
FRI	ES & E		BE			TUTORIALS		

TIME TABLE FOR B.E II YEAR I SEMESTER 2020 - 2021 (ONLINE MODE)								
DEPARTMENT OF CIVIL ENGINEERING								
Branch	Civil II-C							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON	EG		SM		LUNCH BREAK	IP	EM	
TUE	OCE		ES & E			BE	SM	
WED	OCE		SM	SUR & GEO		IP	SUR & GEO	
THUR	ES & E		BE			BE	EG	
FRI	IP		SUR & GEO			TUTORIALS		

11. Individual Time tables

11.1 Individual Time tables (18-19) : Subject : II YR SOM-1-BTECH

Branch	Civil II-A	DEPARTMENT OF CIVIL ENGINEERING						
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:30	1:30 TO 2:20	2:20 TO 3:10	3:10 TO 4:00
MON			SM		LUNCH BREAK			
TUE								SM
WED			SM					
THUR								
FRI								
SAT								

STRENGTH OF MATERIALS - I:MR. YOUSUF

Branch	Civil II-B	DEPARTMENT OF CIVIL ENGINEERING						
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:30	1:30 TO 2:20	2:20 TO 3:10	3:10 TO 4:00
MON					LUNCH BREAK	SM		
TUE								
WED								
THUR						SM		
FRI								
SAT								

STRENGTH OF MATERIALS - I:MR.MUZAMMIL

Branch	Civil II-C	DEPARTMENT OF CIVIL ENGINEERING						
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:30	1:30 TO 2:20	2:20 TO 3:10	3:10 TO 4:00
MON					LUNCH BREAK			
TUE	SM							
WED						SM		
THUR								
FRI			SM					
SAT								

STRENGTH OF MATERIALS - I:MR.FIRASTH ALI

11.2 Individual Time tables (19-20) : Subject : II YR SOM-1-BTECH

DEPARTMENT OF CIVIL ENGINEERING						wef:15-7-2019		
Branch	Civil II-A					G-BLOCK (1st flr) , LH-13		
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON					BREAK LUNCH			
TUE			SM-I					
WED								
THUR	SM-I							
FRI								

STRENGTH OF MATERIALS - I:Mr.JAVID

DEPARTMENT OF CIVIL ENGINEERING						wef:15-7-2019		
Branch	Civil II-B					G-BLOCK (1st flr) , LH-14		
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON					BREAK LUNCH	SM		
TUE								
WED			SM					
THUR								
FRI								

STRENGTH OF MATERIALS - I:Mr.YOUSUF

DEPARTMENT OF CIVIL ENGINEERING						wef:15-7-2019		
Branch	Civil II-C					G-BLOCK (1st flr) , LH-14		
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON					BREAK LUNCH			
TUE								
WED			SM					
THUR						SM		
FRI								

STRENGTH OF MATERIALS - I:Mr.NOOR MOHAMMED

11.2 Individual Time tables (20-21) : Subject : II YR SM-BE

TIME TABLE FOR B.E II YEAR I SEMESTER 2020 - 2021 (ONLINE MODE)								
DEPARTMENT OF CIVIL ENGINEERING								
Branch	Civil II-A							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON					BREAK LUNCH			
TUE			SM					
WED	SM							
THUR								
FRI								

SM : MR. YOUSUF

TIME TABLE FOR B.E II YEAR I SEMESTER 2020 - 2021 (ONLINE MODE)								
DEPARTMENT OF CIVIL ENGINEERING								
Branch	Civil II-B							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON					BREAK LUNCH		SM	
TUE	SM							
WED								
THUR								
FRI								

SM : MR. SHAZEB

TIME TABLE FOR B.E II YEAR I SEMESTER 2020 - 2021 (ONLINE MODE)								
DEPARTMENT OF CIVIL ENGINEERING								
Branch	Civil II-C							
Days	9:30 TO 10:20	10:20 TO 11:10	11:10 TO 12:00	12:00 TO 12:50	12:50 TO 1:40	1:40 TO 2:30	2:30 TO 3:20	3:20 TO 4:10
MON			SM		BREAK LUNCH			
TUE							SM	
WED								
THUR								
FRI								

SM : MR. FIRASATH

12. Instructional Learning Outcomes

S. No.	Topics to be covered	Course Learning Outcomes
	UNIT - I	
1-2	Elasticity and plasticity – Types of stresses and strains – Hooke's law	Understand the concept of elasticity and plasticity, concept of stress and strain, concept of Hooke's law
3-4	Stress – strain diagram for mild steel – Working stress – Factor of safety	Explain Relation between stress and strain for mild steel. Understand working stress and factor of safety.
5-6	Bars of varying section	Understand the concept of bars for varying section.
7-8	Composite bars – Temperature stresses	Understand the concept of composite bars and temperature stresses.
9-10	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	Explain lateral strain, Poisson's ratio, volumetric strain, elastic moduli and relation between them
11-12	Elastic constants.	Explain Bulk modulus, longitudinal strain, lateral strain and relation between them
13-14	Strain Energy, Resilience – Gradual, sudden, impact and shock loadings – simple applications.	Explain resilience, proof resilience, modulus of resilience. Derive strain energy for various loadings and simple
	UNIT - II	
15-16	Definition of beam – Types of beams – Concept of shear force and bending moment	Define beam, types of beams. Explain the concept of shear force and bending moment
17-19	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	Derive and evaluate the shear-force and bending moment for cantilever beam for various types of loading and solved problems
20-22	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of	Derive and evaluate the shear-force and bending moment for simply supported beam for various types of

	contra flexure –	loading and solved problems
23-25	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	Derive and evaluate the shear-force and bending moment for overhanging beam for various types of loading and solved problems
26	Relation between S.F., B.M and rate of loading at a section of a beam.	Explain the relation between shear force and bending moment and rate of loading at a section for beams
27-28	UNIT -III FLEXURAL STRESSES: Theory of simple bending – Assumptions – Derivation of bending equation: $M/I = f/y = E/R$	Explain the concept of simple bending with assumptions and derive the bending equation
29-30	Neutral axis – Determination of bending stresses	Define neutral axis and determine the bending stresses for various conditions
31-32	Section modulus of rectangular and circular sections (Solid and Hollow), I, T, Angle and Channel sections	Derive the section modulus for rectangular, circular, I, T sections and solved problems
33-34	Design of simple beam sections.	Solve problems for design of simple beams
35-38	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.	Derive the formula for shear stress and evaluate the shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.
39	UNIT-IV PRINCIPAL STRESSES AND STRAINS: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	Define principal stresses and strains. Explain the stresses on a inclined section of a bar under axial loading and explain the concept of compound stresses
40-42	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	Evaluate Normal and tangential stresses on an inclined plane for biaxial stresses and evaluate stresses for two perpendicular normal stresses accompanied by a state of simple shear
		Explain the concept of Mohr's circle

42-45	Principal Strains – Analytical and graphical solutions.	of stresses and derive the principal stresses and strains using analytical and graphical method.
46-47	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	Explain Maximum Principal Stress Theory and evaluate failure criteria
48-49	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory (Von Mises Theory).	Explain Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory (Von Mises Theory) and evaluate failure criteria
50-52	UNIT-V DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	Derive relation between slope, deflection and radius of curvature and derive the differential equation for the elastic line of a beam
53-56	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	Determine slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load using Double integration and Macaulay's methods
57-58	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	Explain mohr's theorem and moment area method and apply it to simple beams.
59-62	Conjugate Beam Method: Introduction – Concept of conjugate beam method. Difference between a real beam and a conjugate beam.	Explain conjugate beam method and differentiate between real beam and a conjugate beam.

13. Lecture schedule with methodology being used/adopted

Lesson plan

S. No.	Period No.	Topic	Teaching aids used PPT/ OHP/ BB	Remarks
UNIT-1				
1	2	Elasticity and plasticity – Types of stresses and strains – Hooke's law	BB	
2	2	stress – strain diagram for mild steel – Working stress – Factor of safety	BB	
3	2	Bars of varying section	BB	
4	2	Composite bars – Temperature stresses. Elastic constants.	BB	
5	2	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	BB	
6	2	Elastic Constants	BB	
7	2	Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	BB	
UNIT – 2				
8	2	Definition of beam – Types of beams – Concept of shear force and bending moment	BB	
9	2	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	BB	
10	2	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	BB	
11	2	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	BB	

UNIT – 3				
12	2	FLEXURAL STRESSES: Theory of simple bending – Assumptions –Derivation of bending equation: $M/I = f/y = E/R$	BB	
13	2	Neutral axis – Determination of bending stresses	BB	
14	2	Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections	BB	
15	2	Design of simple beam sections	BB	
16	2	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular,circular, triangular, I, T angle sections.	BB	
UNIT – 4				
17	2	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	BB	
18	2	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	BB	
19	2	Principal Strains – Analytical and graphical solutions	BB	
20	2	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	BB	
21	2	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory(Von Mises Theory).	BB	
UNIT – 5				
22	2	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	BB	
23	2	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	BB	
26	2	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	BB	

24	2	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	BB	
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14. Lesson schedule

**Nawab Shah Alam Khan College of Engineering &
Technology**

Department of Civil Engineering

Lesson Plan & Schedule

14.1 Lesson Plan & Schedule: 18-19

Year & Sem & Sec : II year Sem-I, Sec-A Sub : SOM-1

Faculty Name: MOHD YOUSUF AHMED

S. No.	Date	Topic	Total No. of Periods
		UNIT-1	
1	09-07-18	Elasticity and plasticity – Types of stresses and strains – Hooke's law	2
2	10-07-18 & 11-07-18	stress – strain diagram for mild steel – Working stress – Factor of safety	3
3	16-07-18	Bars of varying section	2
4	17-07-18 & 18-07-18	Composite bars – Temperature stresses. Elastic constants.	3
5	23-07-18	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	2

6	24-07-18	Elastic Constants	1
7	25-07-18	Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	2
		UNIT - 2	
8	30-07-18 & 31-07-18	Definition of beam – Types of beams – Concept of shear force and bending moment	3
9	01-08-18 & 08-08-18	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	4
10	13-08-18 & 14-08-18	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	3
11	20-08-18	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
		UNIT - 3	
12	21-08-18 & 27-08-18	FLEXURAL STRESSES: Theory of simple bending – Assumptions –Derivation of bending equation: $M/I = f/y = E/R$	3
13	28-08-18 & 29-08-18	Neutral axis – Determination of bending stresses	3
14	10-09-18	Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections	2
15	11-09-18	Design of simple beam sections	1
16	12-09-18	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular,circular, triangular, I, T angle sections.	2
		UNIT - 4	
17	17-09-18 & 18-09-18	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	3
18	19-09-18	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	2

19	24-09-18 & 25-09-18	Principal Strains – Analytical and graphical solutions	3
20	26-09-18	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	2
21	1-10-18	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory(Von Mises Theory).	2
		UNIT - 5	
22	3-10-18	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	2
23	08-10-18	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	2
26	09-10-18	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	1
24	10-10-18	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	2

Year&Sem&Sec :II year Sem-I,Sec-B Sub :SOM-1

Faculty Name: MUJJU MUZAMMIL

S. No.	Date	Topic	Total No.of Periods
		UNIT-1	
1	09-07-18	Elasticity and plasticity – Types of stresses and strains – Hooke's law	3
2	12-07-18	stress – strain diagram for mild steel – Working stress – Factor of safety	2
3	16-07-18	Bars of varying section	1.5

4	16-07-18	Composite bars – Temperature stresses. Elastic constants.	1.5
5	19-07-18	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	2
6	23-07-18	Elastic Constants	1.5
7	23-07-18	Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	1.5
		UNIT - 2	
8	26-07-18	Definition of beam – Types of beams – Concept of shear force and bending moment	2
9	30-07-18	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	3
10	02-08-18	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
11	06-08-18	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	3
		UNIT - 3	
12	09-08-18	FLEXURAL STRESSES: Theory of simple bending – Assumptions – Derivation of bending equation: $M/I = f/y = E/R$	2
13	13-08-18	Neutral axis – Determination of bending stresses	3
14	16-08-18	Section modulus of rectangular and circular sections (Solid and Hollow), I, T, Angle and Channel sections	2
15	23-08-18	Design of simple beam sections	2
16	27-08-18	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.	3
		UNIT - 4	

17	30-08-18	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	2
18	10-09-18	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	3
19	17-09-18	Principal Strains – Analytical and graphical solutions	3
20	20-09-18	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	2
21	24-09-18	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory(Von Mises Theory).	3
		UNIT - 5	
22	27-09-18	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	2
23	01-10-18	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	3
26	04-10-18	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	2
24	08-10-18	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	3

Year&Sem&Sec :II year Sem-I,Sec-C

Sub :SOM-1

Faculty Name: MOHD FIRASATH ALI

S. No.	Date	Topic	Total No.of Periods
		UNIT-1	
1	10-07-18	Elasticity and plasticity – Types of stresses and strains – Hooke's law	1
2	11-07-18	stress – strain diagram for mild steel – Working stress – Factor of safety	2
3	13-07-18	Bars of varying section	2
4	17-07-18 & 18-07-18	Composite bars – Temperature stresses. Elastic constants.	3
5	20-07-18	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	2
6	24-07-18	Elastic Constants	1
7	25-07-18	Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	2
		UNIT - 2	
8	31-07-18	Definition of beam – Types of beams – Concept of shear force and bending moment	1
9	01-08-18	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
10	03-08-18	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
11	07-08-18 & 08-08-18	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	3
		UNIT - 3	
12	10-08-18	FLEXURAL STRESSES: Theory of simple bending – Assumptions –Derivation of bending equation: $M/I = f/y = E/R$	2

13	14-08-18	Neutral axis – Determination of bending stresses	1
14	21-08-18	Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections	1
15	28-08-18	Design of simple beam sections	1
16	29-08-18	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular,circular, triangular, I, T angle sections.	2
		UNIT - 4	
17	31-08-18	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	2
18	07-09-18	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	2
19	11-09-18	Principal Strains – Analytical and graphical solutions	1
20	12-09-18	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	2
21	14-09-18	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory(Von Mises Theory).	2
		UNIT - 5	
22	18-09-18 & 19-09-18	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	3
23	25-09-18 & 26-09-18	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	3
24	28-09-18	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	2
25	03-10-18 & 05-10-18	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	4

14.2 Lesson Plan & Schedule: 19-20

Year & Sem&Sec : II year Sem-I, Sec-A Sub : SOM-1

Faculty Name: SHAIK MOHD JAVID

S. No.	Date	Topic	Total No. of Periods
		UNIT-1	
1	16-07-19	Elasticity and plasticity – Types of stresses and strains – Hooke's law	2
2	18-07-19	stress – strain diagram for mild steel – Working stress – Factor of safety	2
3	23-07-19	Bars of varying section	2
4	25-07-19	Composite bars – Temperature stresses. Elastic constants.	2
5	30-07-19	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	2
6	01-08-19	Elastic Constants	2
7	06-08-19	Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	2
		UNIT - 2	
8	08-08-19	Definition of beam – Types of beams – Concept of shear force and bending moment	2
9	13-08-19	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
10	20-08-19	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2

11	22-08-19	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
		UNIT - 3	
12	27-08-19	FLEXURAL STRESSES: Theory of simple bending – Assumptions –Derivation of bending equation: $M/I = f/y = E/R$	2
13	29-08-19	Neutral axis – Determination of bending stresses	2
14	03-09-19 & 05-09-19	Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections	4
15	17-09-19	Design of simple beam sections	2
16	19-09-19	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular,circular, triangular, I, T angle sections.	2
		UNIT - 4	
17	24-09-19	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	2
18	26-09-19	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	2
19	01-10-19	Principal Strains – Analytical and graphical solutions	2
20	03-10-19	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	2
21	22-10-19	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory(Von Mises Theory).	2
		UNIT - 5	
22	24-10-19 & 29-10-19	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	2
23	31-10-19 & 05-11-19	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	2

26	07-11-19	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	2
24	14 -11-19 & 19-11-19	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	2

Year & Sem & Sec : II year Sem-I, Sec-B Sub : SOM-1

Faculty Name: MOHD YOUSUF AHMED

S. No.	Date	Topic	Total No. of Periods
		UNIT-1	
1	15-07-19	Elasticity and plasticity – Types of stresses and strains – Hooke's law	2
2	17-07-19	stress – strain diagram for mild steel – Working stress – Factor of safety	2

3	22-07-19	Bars of varying section	2
4	24-07-19	Composite bars – Temperature stresses. Elastic constants.	2
5	31-07-19	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	2
6	05-08-19	Elastic Constants	2
7	07-08-19	Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	2
		UNIT - 2	
8	14-08-19	Definition of beam – Types of beams – Concept of shear force and bending moment	2
9	19-08-19	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
10	21-08-19	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
11	26-08-19	S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
		UNIT - 3	
12	04-09-19	FLEXURAL STRESSES: Theory of simple bending – Assumptions –Derivation of bending equation: $M/I = f/y = E/R$	2
13	09-09-19	Neutral axis – Determination of bending stresses	2
14	16-09-19	Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections	2
15	16-09-19	Design of simple beam sections	2
16	18-09-19	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular,circular, triangular, I, T angle sections.	2

		UNIT - 4	
17	23-09-19	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	2
18	25-09-19	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	2
19	30-09-19	Principal Strains – Analytical and graphical solutions	2
20	21-10-19	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	2
21	23-10-19	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory(Von Mises Theory).	2
		UNIT - 5	
22	28-10-19 & 30-10-19	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	4
23	04-11-19	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	2
26	06-11-19	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	2
24	13-11-19 & 18-11-19	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	4

Year & Sem & Sec : II year Sem-I, Sec-C Sub : SOM-1

Faculty Name: NOOR MOHAMMED

S. No.	Date	Topic	Total No. of Periods
		UNIT-1	
1	17-07-19	Elasticity and plasticity – Types of stresses and strains – Hooke's law	2
2	18-07-19	stress – strain diagram for mild steel – Working stress – Factor of safety	2
3	24-07-19	Bars of varying section	2
4	25-07-19	Composite bars – Temperature stresses. Elastic constants.	2
5	31-07-19	Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them	2
6	07-08-19	Elastic Constants	2
		Strain Energy, Resilience-Gradual, sudden, impact and shock loadings - simple applications	2
8	08-08-19	UNIT - 2	
9	14-08-19	Definition of beam – Types of beams – Concept of shear force and bending moment	2
10	21-08-19	S.F and B.M diagrams for cantilever, subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
11	22-08-19	S.F and B.M diagrams simply supported subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2

		S.F and B.M diagrams for overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contra flexure	2
		UNIT - 3	
12	28-08-19	FLEXURAL STRESSES: Theory of simple bending – Assumptions – Derivation of bending equation: $M/I = f/y = E/R$	2
13	29-08-19	Neutral axis – Determination of bending stresses	2
14	04-09-19	Section modulus of rectangular and circular sections (Solid and Hollow), I, T, Angle and Channel sections	4
15	05-09-19	Design of simple beam sections	2
16	11-09-19	SHEAR STRESSES: Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.	2
		UNIT - 4	
17	18-09-19	Principal stresses and Strains: Introduction – Stresses on an inclined section of a bar under axial loading – compound stresses	2
18	19-09-19	Normal and tangential stresses on an inclined plane for biaxial stresses – Two perpendicular normal stresses accompanied by a state of simple shear - Mohr's circle of stresses, Principal stresses	2
19	25-09-19	Principal Strains – Analytical and graphical solutions	2
20	26-10-19	THEORIES OF FAILURE: Introduction – Various theories of failure - Maximum Principal Stress Theory	2
21	03-10-19	Maximum Principal Strain Theory, Strain Energy and Shear Strain Energy Theory (Von Mises Theory).	2
		UNIT - 5	
22	23-10-19 & 24-10-19	DEFLECTION OF BEAMS: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam	2
23	30-10-19 & 31-10-19	Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, U.D.L, Uniformly varying load	4

26	06-11-19 & 07-11-19	Mohr's theorems – Moment area method – application to simple cases including overhanging beams.	2
24	13-11-19 & 1-11-19	Conjugate Beam Method: Introduction – Concept of conjugate beam method Difference between a real beam and a conjugate beam.	4

14.3 Lesson Plan & Schedule: 20-21

Faculty Name: MOHD YOUSUF AHMED

Year & Sem & Sec : II year Sem-I, Sec-A Sub : SM

S. No.	Date	Topic	Total No. of Periods
		UNIT-1	
1	01-09-20	Definitions of stresses and strains, Hooke's Law,	2
2	02-09-20 & 08-09-20	Modulus of Elasticity, Stress - Strain curve for ductile materials	4
3	09-09-20	Elastic constants, compound bars and temperature stresses.	2
4	15-09-20 & 16-09-20	Strain Energy: Strain energy and resilience in statically determinate bars	4
5	22-09-20	Determinate bars subjected to gradually applied, suddenly applied,	2
6	23-09-20	Determinate bars subjected to impact and shock loads.	2

		UNIT - 2	
7	29-09-20 & 30-09-20	Compound Stresses: Stresses on oblique planes, principal stresses and planes. Mohr's circle of stress.	4
8	06-10-20 & 06-10-20	Theories of Failure based on maximum principal stress, maximum principal strain, maximum shear stress.	4
9	07-10-20	maximum strain energy and maximum shear strain energy	2
10	13-10-20 & 14-10-20	Application to pressure vessels: Thin cylinders subjected to internal fluid pressure, volumetric change.	4
11	20-10-20 & 21-10-20	Thick Cylinders: Lamé's equations, stresses under internal and external fluid pressures, Compound cylinders, Shrink fit pressure.	4
		UNIT - 3	
12	27-10-20 & 03-11-20	Shear Force and Bending Moment: Different types of beams and loads	4
13	10-11-20	Shear force and bending moment diagrams for cantilever, and simply supported beams with and without over hangs.	2
14	11-11-20	Shear force and bending moment diagrams for over hangs subjected to different kinds of loads viz., point loads, uniformly distributed loads, uniformly varying loads and couples.	2
15	17-11-20 & 18-11-20	Bending Stresses in Beams: Assumptions in theory of simple bending, Derivation of flexure equation	4
16	24-11-20 & 25-11-20	Moment of resistance, calculation of stresses in statically determinate beams for different loads and different types of structural sections.	4
		UNIT - 4	
17	01-12-20 & 02-12-20	Shear Stress in Beams: Derivation of equation of shear stresses,	4
18	08-12-20 & 09-12-20	Shear Stress in Beams: Distribution across rectangular Section	4
19	15-12-20 & 16-12-20	Shear Stress in Beams: Distribution across, circular, T and I section.	4

20	22-12-20 & 23-12-20	Direct and Bending Stresses: Direct loading, Eccentric loading, limit of eccentricity.	4
21	29-12-20 & 30-12-20	Core of sections, rectangular and circular, solid and hollow sections.	4
		UNIT - 5	
22	05-01-21 & 06-01-21	Torsion: Theory of pure torsion in solid and hollow circular shafts, shear stress, angle of twist	4
23	12-01-21 & 19-01-21	strength and stiffness of shafts, Transmission of Power. Combined torsion and bending with and without end thrust.	4
24	20-01-21 & 27-01-21	Determination of principal stresses and maximum shear stress. Equivalent bending moment and equivalent twisting moment.	4
25	02-02-21 & 03-02-21	Springs: Close and open coiled helical springs under axial load and axial twist, Carriage springs.	4

Faculty Name: MOHD MUBASHEER SHAZEB

Year & Sem & Sec : II year Sem-I, Sec-B

Sub : SM

S. No.	Date	Topic	Total No. of Periods
		UNIT-1	
1	01-09-20	Definitions of stresses and strains, Hooke's Law,	2
2	07-09-20	Modulus of Elasticity, Stress - Strain curve for ductile materials	4
3	08-09-20 & 14-09-20	Elastic constants, compound bars and temperature stresses.	2
4	15-09-20 & 21-09-20	Strain Energy: Strain energy and resilience in statically determinate bars	4
5	22-09-20	Determinate bars subjected to gradually applied, suddenly applied,	2
6	28-09-20	Determinate bars subjected to impact and shock loads.	2
		UNIT - 2	
7	29-09-20	Compound Stresses: Stresses on oblique planes, principal stresses and planes. Mohr's circle of stress.	4
8	05-10-20 & 06-10-20	Theories of Failure based on maximum principal stress, maximum principal strain, maximum shear stress.	4
9	12-10-20	maximum strain energy and maximum shear strain energy	2
10	13-10-20 & 19-10-20	Application to pressure vessels: Thin cylinders subjected to internal fluid pressure, volumetric change.	4
11	20-10-20 & 26-10-20	Thick Cylinders: Lamé's equations, stresses under internal and external fluid pressures, Compound cylinders, Shrink fit pressure.	4
		UNIT - 3	

12	27-10-20 & 02-11-20	Shear Force and Bending Moment: Different types of beams and loads	4
13	03-11-20	Shear force and bending moment diagrams for cantilever, and simply supported beams with and without over hangs.	2
14	09-11-20 & 10-11-20	Shear force and bending moment diagrams for over hangs subjected to different kinds of loads viz., point loads, uniformly distributed loads, uniformly varying loads and couples.	2
15	16-11-20 & 17-11-20	Bending Stresses in Beams: Assumptions in theory of simple bending, Derivation of flexure equation	4
16	23-11-20 & 24-11-20	Moment of resistance, calculation of stresses in statically determinate beams for different loads and different types of structural sections.	4
		UNIT - 4	
17	01-12-20 & 07-12-20	Shear Stress in Beams: Derivation of equation of shear stresses,	4
18	08-12-20 & 14-12-20	Shear Stress in Beams: Distribution across rectangular Section	4
19	15-12-20 & 21-12-20	Shear Stress in Beams: Distribution across, circular, T and I section.	4
20	22-12-20 & 28-12-20	Direct and Bending Stresses: Direct loading, Eccentric loading, limit of eccentricity.	4
21	29-12-20 & 04-01-21	Core of sections, rectangular and circular, solid and hollow sections.	4
		UNIT - 5	
22	05-01-21 & 11-01-21	Torsion: Theory of pure torsion in solid and hollow circular shafts, shear stress, angle of twist	4
23	12-01-21 & 18-01-21	strength and stiffness of shafts, Transmission of Power. Combined torsion and bending with and without end thrust.	4
24	25-01-21 & 02-02-21	Determination of principal stresses and maximum shear stress. Equivalent bending moment and equivalent twisting moment.	4
25	08-02-21 & 09-02-21	Springs: Close and open coiled helical springs under axial load and axial twist, Carriage springs.	4

Faculty Name: MOHD FIRASATH ALI

Year&Sem&Sec : II year Sem-I, Sec-C Sub : SM

	Date	Topic	
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S. No.			Total No. of Periods
		UNIT-1	
1	01-09-20	Definitions of stresses and strains, Hooke's Law,	2
2	07-09-20	Modulus of Elasticity, Stress - Strain curve for ductile materials	2
3	08-09-20 & 14-09-20	Elastic constants, compound bars and temperature stresses.	4
4	15-09-20 & 21-09-20	Strain Energy: Strain energy and resilience in statically determinate bars	4
5	22-09-20	Determinate bars subjected to gradually applied, suddenly applied,	2
6	28-09-20	Determinate bars subjected to impact and shock loads.	2
		UNIT - 2	
7	29-09-20	Compound Stresses: Stresses on oblique planes, principal stresses and planes. Mohr's circle of stress.	2
8	05-10-20 & 06-10-20	Theories of Failure based on maximum principal stress, maximum principal strain, maximum shear stress.	4
9	12-10-20	maximum strain energy and maximum shear strain energy	2
10	13-10-20 & 19-10-20	Application to pressure vessels: Thin cylinders subjected to internal fluid pressure, volumetric change.	4
11	20-10-20 & 26-10-20	Thick Cylinders: Lamé's equations, stresses under internal and external fluid pressures, Compound cylinders, Shrink fit pressure.	4
		UNIT - 3	
12	27-10-20 & 02-11-20	Shear Force and Bending Moment: Different types of beams and loads	4

13	03-11-20	Shear force and bending moment diagrams for cantilever, and simply supported beams with and without over hangs.	2
14	09-11-20 & 10-11-20	Shear force and bending moment diagrams for over hangs subjected to different kinds of loads viz., point loads, uniformly distributed loads, uniformly varying loads and couples.	4
15	16-11-20 & 17-11-20	Bending Stresses in Beams: Assumptions in theory of simple bending, Derivation of flexure equation	4
16	23-11-20 & 24-11-20	Moment of resistance, calculation of stresses in statically determinate beams for different loads and different types of structural sections.	4
		UNIT - 4	
17	01-12-20 & 07-12-20	Shear Stress in Beams: Derivation of equation of shear stresses,	4
18	08-12-20 & 14-12-20	Shear Stress in Beams: Distribution across rectangular Section	4
19	15-12-20 & 21-12-20	Shear Stress in Beams: Distribution across, circular, T and I section.	4
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21	29-12-20 & 04-01-21	Core of sections, rectangular and circular, solid and hollow sections.	4
		UNIT - 5	
22	05-01-21 & 11-01-21	Torsion: Theory of pure torsion in solid and hollow circular shafts, shear stress, angle of twist	4
23	12-01-21 & 18-01-21	strength and stiffness of shafts, Transmission of Power. Combined torsion and bending with and without end thrust.	4
24	25-01-21 & 02-02-21	Determination of principal stresses and maximum shear stress. Equivalent bending moment and equivalent twisting moment.	4
25	08-02-21 & 09-02-21	Springs: Close and open coiled helical springs under axial load and axial twist, Carriage springs.	4

15. University question Papers of Previous Years



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R16

Code No: 133BT

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2018

STRENGTH OF MATERIALS - I
(Common to CE, CEE)

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART - A

(25 Marks)

1. a) Distinguish between Tensile stress and Compressive strain. [2]
- b) Draw the stress strain diagram for mild steel and identify the significant points. [3]
- c) Draw the SFD and BMD for a cantilever beam of length L subjected to udl w per unit length. [2]
- d) List any three important points to be kept in mind while drawing SFD and BMD. [3]
- e) Define Neutral Axis and Moment of Resistance for a beam. [2]
- f) List the assumptions made in the theory of simple bending. [3]
- g) List the cases where Mohr's theorem is conveniently used. [2]
- h) A rectangular bar of cross sectional area 10000mm^2 is subjected to an axial load of 25kN. Determine the normal stress on a section which is inclined at 30° with normal cross section of the bar. [3]
- i) Define principal stresses and strains. [2]
- j) What is meant by Mohr's circle of stresses? [3]

PART-B

(50 Marks)

2. A reinforced concrete column $500\text{mm} \times 500\text{mm}$ has Four Reinforcement bars of Steel each 18 mm in diameter one in each corner. Find the stresses in concrete and steel bars when the column is subjected to a load of 2MN. Take E for steel is $2.1 \times 10^5 \text{ N/mm}^2$ and for concrete as $1.4 \times 10^4 \text{ N/mm}^2$. [10]

OR

3. A steel rod of 20mm diameter passes centrally through a copper tube of 50mm external diameter and 40mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly on the projecting parts of the rod. If the temperature of the assembly is raised by 50°C , calculate the stresses developed in copper and steel. Take E for steel and copper as 200GN/m^2 and 100GN/m^2 and α for steel and copper as $12 \times 10^{-6} \text{ per } ^\circ\text{C}$ and $18 \times 10^{-6} \text{ per } ^\circ\text{C}$. [10]
4. A simply supported beam of length 12m, carries the uniformly distributed load of 10kN/m over a length of 4m starting from 4m from the left support. Point loads of 50kN and 40kN acts at a distance of 4m and 8m from the left support. Draw the S.F and B.M diagrams for the beam. Also calculate the maximum bending moment. [10]

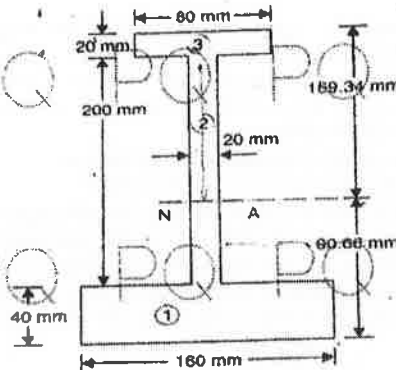
OR



5. A cantilever beam of length 2m carries the point loads 200N, 400N and 700N at distances 0.5m, 1.2m and 2m respectively from the fixed end. Draw the SF and BM diagrams for cantilever beam. [10]

- 6.a) A steel plate of width 100mm and of thickness 18mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E=2 \times 10^5 \text{ N/mm}^2$.
b) A rectangular beam 100mm wide and 250mm deep is subjected to a maximum shear force of 50kN. Determine Average shear stress, maximum shear stress and shear stress at a distance of 25mm above the neutral axis. [5+5]

7. OR
A cast iron beam is of I-Section is as shown in Figure. The beam is simply supported on a span of 5 meters. If the tensile stress is not to exceed 20 N/mm^2 , find the safe uniformly load which the beam can carry. Find also the maximum compressive stress and draw bending stress distribution of the section and locate the stresses. [10]



8. Derive the deflection equation for a simply supported beam of length L carrying a point load W at the centre. [10]

9. OR
A simply supported beam of length 4m carries a point load of 3kN at a distance of 1m from each end. Take $E=2 \times 10^5 \text{ N/mm}^2$ and $I=10^8 \text{ mm}^4$ for the beam. Using conjugate beam method determine (a) Slope at each end and under each load. (b) Deflection under each load and at the center. [10]

- 10.a) At a point in a strained material, the principal stresses are 400 N/mm^2 and 300 N/mm^2 . The first one is tensile in nature and the second one is compressive in nature. Determine the following stresses on a plane inclined at 60° to the direction of the larger stress. (i) Normal stress. (ii) Shear stress. (iii) Resultant stress.
b) A rectangular bar of cross sectional area 10000 mm^2 is subjected to a tensile load of P . The permissible normal and shear stresses on the oblique plane which is inclined at 60° are 8 N/mm^2 and 8 N/mm^2 . Determine the safe value of P . [5+5]

11. OR
Discuss in detail various prominent theories of failures. [10]

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Code No: 133BT

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, April/May - 2018

STRENGTH OF MATERIALS - I

(Common to CE, CEE)

Time: 3 Hours

Max. Marks: 75

AG AG AG AG AG AG AG A

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each full-question carries 10 marks.

AG AG AG AG AG AG AG A

PART-A

(25 Marks)

- 1.a) A circular steel bar of length ' L ' cross-sectional area ' A ' and weight ' W ' is fixed at its upper end and hangs vertically. Find the elongation of the bar under its own weight. [2]
- b) A steel bar of diameter 20 mm and gauge length 100 mm is tested in UTM. Find the change in the diameter of the bar at 100 kN load if the Poisson's ratio is 0.25 and modulus of elasticity $= 2 \times 10^5 \text{ N/mm}^2$. [3]
- c) Define the shear force and bending moment at a cross-section. [2]
- d) Obtain the relation between shear force and rate of loading. [3]
- e) A beam has a circular cross-section of diameter 300 mm and subjected to a shear force of 240 kN. Determine the ratio of average shear stress to the maximum shear stress. [2]
- f) A simply supported beam has a T-section as shown in Figure 1 and carries uniformly distributed load. Determine the bending stress at the extreme top fibre if the stress at the extreme bottom fibre is 125 N/mm^2 . The depth of the web of T-section is 150 mm. [3]

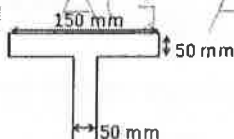


Figure 1

- g) Distinguish between a real beam and a conjugate beam. [2]
- h) A simply supported beam of span 3 m is subjected to a concentrated load of 50 kN at its mid-span. Determine the flexural rigidity if the maximum deflection is 20 mm. [3]
- i) What is the importance of Mohr's circle of stress? [2]
- j) Explain maximum principal strain theory of failure. [3]

PART-B

(50 Marks)

2. A solid steel bar 900 mm long and 75 mm diameter is placed concentrically inside an aluminium tube having 80 mm inside diameter and 100 mm outside diameter. The aluminium tube is 0.25 mm longer than the steel bar. Find the stresses in the steel bar and aluminium tube, if an axial compressive load of 600 kN is applied to the bar and tube through rigid cover plates attached at both ends. $E_{\text{Steel}} = 200 \text{ GPa}$ and $E_{\text{Aluminium}} = 70 \text{ GPa}$. [10]

AG AG AG AG AG AG AG A

OR

- 3.a) Derive the relation between the modulus of elasticity and bulk modulus.
b) A steel rod of length 1.25 m and 22 mm diameter hangs vertically with a collar firmly attached at the lower end of the rod. Find the maximum stress induced in the rod when a block of weight 25 kg falls on the collar from a clear height of 300 mm. Also find the energy absorbed and the modulus of resilience. Use modulus of elasticity = $2 \times 10^5 \text{ N/mm}^2$. [5+5]
4. Draw the shear force and bending moment diagrams for a beam supported and loaded as shown in Figure 2. [10]

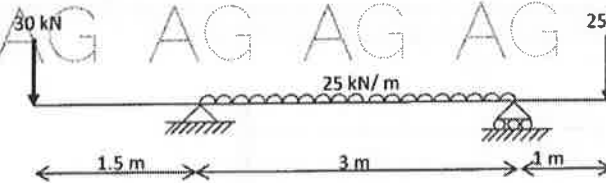


Figure 2

OR

5. Draw the shear force and bending moment diagrams for a beam of span 5 m supported and loaded as shown in Figure 3. [10]

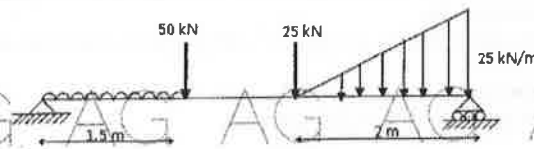


Figure 3

6. A beam of I-section has top flange $125 \text{ mm} \times 16 \text{ mm}$, bottom flange $150 \text{ mm} \times 20 \text{ mm}$ and web of thickness 12 mm. The total depth of the beam is 250 mm and simply supported over a span of 5 m. The beam is subjected to uniformly distributed load of 50 kN/m over its entire span in addition to a concentrated load 60 kN at its mid-span. Draw the bending stress distribution across the depth of the beam cross-section at a section located 3 m from the left support. [10]

OR

7. A steel beam of depth 250 mm has cross-section as shown in Figure 4. The beam section is subjected to a shear force of 150 kN. Draw the shear stress distribution across the depth of the section. Also determine ratio of maximum shear stress to the mean shear stress. [10]

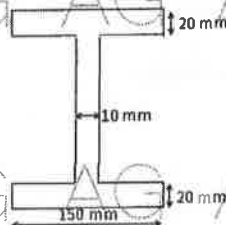


Figure 4

8. Determine the maximum deflection and the slopes at the supports of a beam supported and loaded as shown in Figure 5. [10]

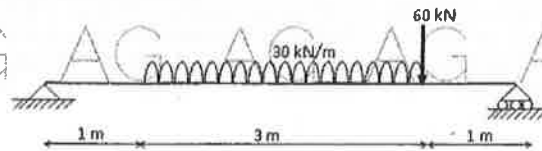


Figure 5
OR

9. Using the conjugate beam method, determine the maximum deflection and the slope at the free end of a beam supported and loaded as shown in Figure 6. [10]

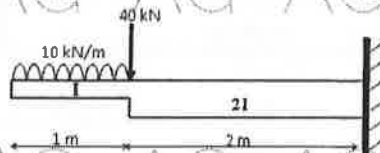


Figure 6

10. The state of stress at a point of a loaded member is shown in Figure 7, using the Mohr's circle of stresses, determine the
a) Stresses acting on a plane making an angle 30° with respect to horizontal in clock-wise direction
b) Magnitude of the maximum shear stress and
c) Magnitude and the direction of principal stresses. [10]

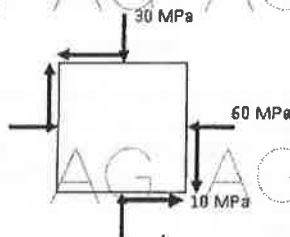


Figure 7
OR

11. Explain the following theories of failure:
a) Maximum principal stress theory and
b) Von-Mises theory. [5+5]

—ooOoo—

16. QUESTION BANK AND ASSIGNMENT QUESTIONS

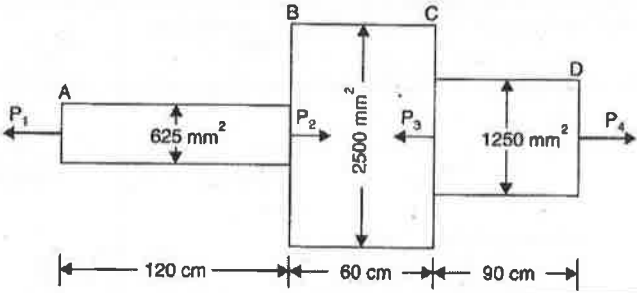
S No	QUESTION	Blooms taxonomy level	Course Outcomes
UNIT - I			
SIMPLE STRESSES AND STRAINS, STRAIN ENERGY			
Part - A (Short Answer Questions)			
1	Distinguish between the terms (a) Elasticity and (b) Plasticity with examples.	Remembering	1
2	Define the following properties of engineering materials: (a) Ductility (b) Brittleness (c) Malleability	Remembering	1
3	Define the following properties of engineering materials: (a) Toughness (b) Hardness (c) Strength	Remembering	1
4	Define Stress at a point in a material, and mention its units.	Remembering	1
5	Distinguish between different types of stress using illustrations	Remembering	1
6	Define Strain in a material and give its units	Remembering	1
7	State Hooke's law and give its equation	Remembering	1
8	Distinguish between different types of strain	Remembering	1
9	Define modulus of elasticity and give its units.	Remembering	1
10	Draw stress-strain diagram for mild steel indicating all critical points	Understanding	1
11	Define longitudinal strain and lateral strain.	Remembering	1
12	Define Poisson's ratio and its range of values	Remembering	1

	Define Volumetric strain and bulk modulus	Remembering	1
14	Give the relationship between Young's modulus, Rigidity Modulus and Bulk Modulus.	Remembering	1
15	Define rigidity modulus and give its units	Remembering	1
16	What is meant by strain energy?	Understanding	1
17	Distinguish between modulus of resilience and modulus of toughness.	Understanding	1
18	Define resilience and proof resilience.	Understanding	1
19	What is working stress?	Understanding	1
20	Define factor of safety and state why it is used?	Understanding	1
Part - B (Long Answer Questions)			
1	Explain with illustrations and stress-strain diagrams, the phenomenon of strain-hardening.	Understanding	1
2	Explain with illustrations and stress-strain diagrams, the phenomenon of necking.	Understanding	1
3	Define and explain the terms: slip and creep.	Understanding	1
4	Explain the off-set method of locating the yield point for a material on its stress-strain curve.	Understanding	1
5	Explain the concept of fatigue failure. Define endurance limit and fatigue limit.	Understanding	1
6	A tensile test was conducted on a mild steel bar. The following data was obtained from the test: Diameter of steel bar = 2.5 cm; Gauge length of the bar = 24 cm;	Applying	1,2

	Diameter of the bar at rupture = 2.35 cm; Gauge length at rupture = 24.92mm Determine (a) percentage elongation (b) percentage decrease in area		
7	A tensile test was conducted on a mild steel bar. The following data was obtained from the test: Diameter of steel bar = 3cm; Gauge length of the bar = 20cm Load at elastic limit = 250kN; Extension at load of 150kN = 0.21mm Maximum load = 380kN; Determine: (a) Young's modulus (b) Yield strength (c) Ultimate Strength (d) Strain at the elastic limit	Applying	1,2
8	A steel bar of 25 mm diameter is tested in tension and following is observed: Limit of Proportionality = 196.32 kN; Load at yield = 218.13 kN, Ultimate load = 278.20 kN. Compute the stresses in the specimen at various stages. If the factor of safety is 1.85, determine the permissible stress in the material.	Applying	1,2
9	A steel bar of 25 mm diameter was tested in tension and following were observed: Limit of Proportionality = 196.32 kN; Load at yield = 218.13 kN, Ultimate load = 278.20 kN. At the proportional limit, the elongation measured over a gauge length of 100 mm was 0.189 mm. After fracture, the length between the gauge points was 112.62 mm and the minimum diameter was 23.64. Determine the Young's modulus and measures of ductility (percentage elongation and percentage contraction),	Applying	1,2
10	A 3.5 m long steel column of cross-sectional area 5000 mm^2 , is subjected to a load of 1.6 MN. Determine the factor of safety for the column, if the yield stress of steel is 550 MPa. Determine the allowable load on the column, if the deformation of the column should not exceed 5.0 mm. Assume Young's modulus of steel as 195 GPa.	Applying	1,2
11	A 2.0 m long steel tie bar is subjected to force of 150 kN. Determine its cross-section so that (i) the stress does not exceed 140 MPa (ii) the extension is not more than 1.2 mm. Assume Young's modulus of 210 GPa. If steel bars are	Applying	1,2

	available in increments of 5 mm from 30 mm diameter onwards, choose the appropriate diameter for both cases.		
12	Design a steel rod to sustain a load of 80 kN with a safety factor 2.5. What is the maximum permissible length of the rod, if the allowable deformation is 0.5 mm? Assume a yield stress of 230 MPa and Young's modulus of 195 GPa.	Applying	1,2
13	Derive the constitutive relationship between stress and strain for three dimensional stress systems.	Applying	1,2
14	A rod whose ends are fixed to rigid supports, is heated so that rise in temperature is $T^{\circ}\text{C}$. Derive the expression for thermal strain and thermal stresses set up in the body if α is co-efficient of thermal expansion.	Applying	1,2
15	Derive the expression for volumetric strain of a body in terms of its linear strains in orthogonal directions.	Applying	1,2
16	Derive relationships between Young's modulus, rigidity modulus and bulk modulus, including Poisson's ratio into the relationships.	Applying	1,2
17	Determine the Poisson's ratio and bulk modulus of a material, for which Young's modulus is $1.2 \times 10^5 \text{ N/mm}^2$ and modulus of rigidity is $4.5 \times 10^4 \text{ N/mm}^2$	Applying	1,2
18	Prove that maximum strain energy stored per unit volume in a body is given by —	Applying	1,2
19	If the extension produced in a rod due to impact load is very small in comparison with the height through which the load falls, prove that stress induced in the body is given by — —	Applying	1,2
20	Prove that the stress developed in a body due to load P when it is applied suddenly is given by	Applying	1,2

Part - C (Problem Solving and Critical Thinking Questions)

1	A tensile test was conducted on a mild steel bar. The following data was obtained from the test: Diameter of steel bar = 3 cm Gauge length of the bar = 20 cm Load at elastic limit = 250 kN Extension at load of 150 kN = 0.21 mm Maximum load = 380 kN Total extension = 60 mm Diameter of rod at failure = 2.25 cm Determine: (a) Young's modulus (b) stress at elastic limit (c) percentage elongation (d) percentage decrease in area	Analyze & evaluate	1,2,3
2	A member ABCD is subjected to point loads P_1, P_2, P_3 and P_4 as shown in figure below. Calculate the force P_2 necessary for equilibrium, if $P_1 = 45\text{kN}$, $P_2 = 450\text{kN}$ and $P_4 = 130\text{kN}$. Determine the total elongation of the member, assuming the modulus of elasticity to be $2.1 \times 10^5 \text{ N/mm}^2$. 	Analyze & evaluate	1,2,3

A compound tube consists of a steel tube 140mm internal diameter and 160mm

3	external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900kN. Find the stresses and the load carried by each tube and the amount it shortens. Length of each tube is 140mm. Take E for steel as $2 \times 10^5 \text{ N/mm}^2$ and for brass as $1 \times 10^5 \text{ N/mm}^2$.	Analyze & evaluate	1,2,3
4	A steel rod of 3cm diameter and 5m long is connected to two grips and the rod is maintained at a temperature of 95°C . Determine the stress and pull exerted when the temperature falls to 30°C , if (i) the ends do not yield, and (ii) the ends yield by 0.12cm. Take $E = 2 \times 10^5 \text{ MN/m}^2$ and $\alpha = 12 \times 10^{-6}/^\circ\text{C}$.	Analyze & evaluate	1,2,3
5	Determine the value of Young's modulus and Poisson's ratio of a metallic bar of length 25cm, breadth 3cm and depth 2cm when the bar is subjected to an axial compressive load of 240kN. The decrease in length is given as 0.05cm and increase in breadth is 0.002cm.	Analyze & evaluate	1,2,3
6	A metallic block 250mm x 80mm x 30mm is subjected to a tensile force of 20kN, 30kN and 15kN along x, y and z directions respectively. Determine the change in volume of the block. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.30.	Analyze & evaluate	1,2,3
7	Determine the Poisson's ratio and bulk modulus of a material, for which Young's modulus is $1.2 \times 10^5 \text{ N/mm}^2$ and modulus of rigidity is $4.5 \times 10^4 \text{ N/mm}^2$	Analyze & evaluate	1,2,3
8	A bar of 30mm in diameter was subjected to tensile load of 54kN and the measured extension on 300mm gauge length was 0.112mm and change in diameter was 0.0036mm. Calculate the Poisson's ratio and three Moduli.	Analyze & evaluate	1,2,3
9	A bar of uniform cross-section 'A' and length 'L' hangs vertically, subjected to its own weight. Prove that the strain energy stored within the bar is given by	Analyze & evaluate	1,2,3
	A vertical round steel rod 1.82m long is securely held at its upper end. A weight can slide freely on the rod and its fall is arrested by a stop provided at		

10	the lower end of the rod. When the weight falls from a height of 30mm above the stop, the maximum stress reached in the rod is estimated to be 157N/mm^2. Determine the stress if the load has been applied gradually and also the maximum stress if the load had fallen from a height of 47.5mm. Take $E = 2.1 \times 10^5 \text{ N/mm}^2$.	Analyze & evaluate	1,2,3
11	A bar of length l has its diameter increasing from d at one end to D at the other. Determine the deformation of the member subjected to a tensile force of P.	Analyze & evaluate	1,2,3
12	A prismatic bar of length l and unit weight w is suspended freely from its end. Determine the elongation of the member under gravity.	Analyze & evaluate	1,2,3
13	A concrete column is reinforced with steel bars comprising 6 percent of the gross area of the column section. What is the fraction of the compressive load sustained by steel bars, if the ratio of Young's moduli of steel and concrete is 12.5?	Analyze & evaluate	1,2,3
14	A steel rod of uniform square cross-section with side 22.0 mm and length 500.00 mm is rigidly held between its end by fixed supports. Assuming $\alpha = 12.5 \times 10^{-6}$ per K and $E = 150.0 \text{ GPa}$, determine the force and the stress in the rod when it is subjected to (i) rise in temperature of 85 K and (ii) fall in temperature of 65 K?	Analyze & evaluate	1,2,3
15	A steel rod of tapered square cross-section with larger side 40 mm and smaller side 20 mm and length 650 mm is rigidly held between its end by fixed supports. Assuming $\alpha = 12.5 \times 10^{-6}$ per K and $E = 150.0 \text{ GPa}$, determine the force in the rod when it is subjected to (i) rise in temperature of 85 K and (ii) fall in temperature of 65 K?	Analyze & evaluate	1,2,3
16	A steel rod of tapered circular cross-section with larger end diameter 65 mm and smaller end diameter 33 mm and length 810 mm is rigidly held between its end by fixed supports. Assuming $\alpha = 12.5 \times 10^{-6}$ per K and $E = 150.0 \text{ GPa}$,	Analyze &	1,2,3

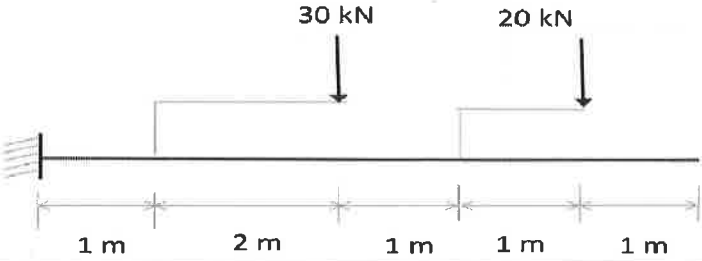
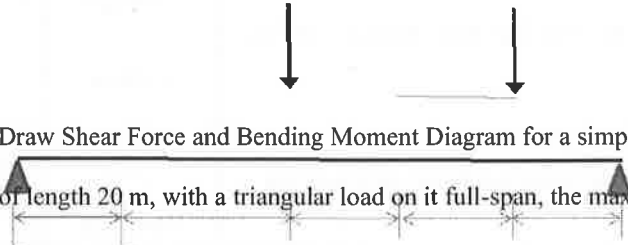
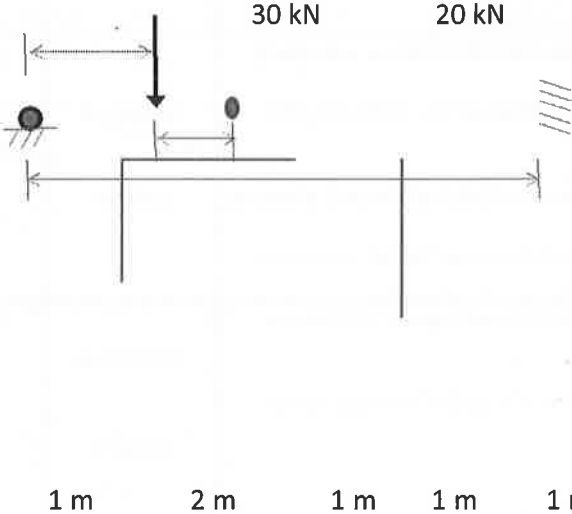
	determine the force in the rod when it is subjected to (i) rise in temperature of 85 K and (ii) fall in temperature of 65 K?	evaluate	
17	A compound bar comprises of a 12.5 mm diameter aluminum rod and a copper tube of inner diameter 14.5 mm and outer diameter 25 mm. If the Young's moduli of aluminum and copper are 80 GPa and 120 GPa, respectively, then determine the stress in the assembly when subjected to (i) a temperature rise of 95 K, and (ii) a temperature fall of 35 K. Take $\alpha = 14.6 \times 10^{-6}$ per K for aluminum and $\alpha = 16.8 \times 10^{-6}$ per K for copper.	Analyze & evaluate	1,2,3
18	Determine the resilience and toughness moduli of mild steel ($E = 200$ GPa) with a yield stress of 250 MPa and fracture strain of 28.5 percent. Neglect strain hardening effects. From these data determine the impact resistance of a bar of 12 mm diameter and 500 mm length.	Analyze & evaluate	1,2,3
19	A mass of 250 kg falls through a height of 300 mm on a concrete column of 230 x 500 mm section. Determine the maximum stress and deformation in the 4.5m long column, if the Young's modulus of concrete is 20 GPa.	Analyze & evaluate	1,2,3
20	Compute the strain energy in a steel bar ($E = 200$ GPa) of length 2.7 m and 22 mm diameter under a load of 50 kN. What is the resilience modulus of the bar, if the yield stress is 240 MPa?	Analyze & evaluate	1,2,3
UNIT - II			
SHEAR FORCE AND BENDING MOMENT			
Part – A (Short Answer Questions)			
1	What are the different types of beams?	Remembering	2
2	Differentiate between a simply supported beam and a cantilever.	Remembering	2

3	Differentiate between a fixed beam and a cantilever.	Remembering	2
4	Show by proper diagram, positive and negative shear forces at a section of a beam.	Remembering	2
5	Draw shear force diagrams for a cantilever of length L carrying a point load W at the free end.	Applying	2,4
6	Draw shear force diagrams for a cantilever of length L carrying a point load W at the mid-span.	Applying	2,4
7	Draw shear force diagram for a cantilever of length L carrying a uniformly distributed load of w per unit length over its entire span.	Applying	2,4
8	Draw shear force diagrams for a simply supported beam of length L carrying a point load W at its mid-span.	Applying	2,4
9	Draw shear force diagram for a simply supported beam of length L carrying a uniformly distributed load of w per unit length over its entire span.	Applying	2,4
10	Explain what information we obtain from shear force diagram and bending moment diagram.	Understanding	2,4
11	Draw bending moment diagrams for a cantilever of length L carrying a point load W at the free end.	Applying	2,4
12	Draw bending moment diagram for a cantilever of length L carrying a point load W at the mid-span.	Applying	2,4
13	Draw bending moment diagram for a cantilever of length L carrying a uniformly distributed load of w per unit length over its entire span.	Applying	2,4
14	Draw bending moment diagram for a simply supported beam of length L carrying a point load W at its mid-span.	Applying	2,4
15	Draw bending moment diagram for a simply supported beam of length L carrying a uniformly distributed load of w per unit length over its entire span.	Applying	2,4
16	Draw bending moment diagram for a cantilever beam of length L with a positive moment M applied at its free end.	Applying	2,4

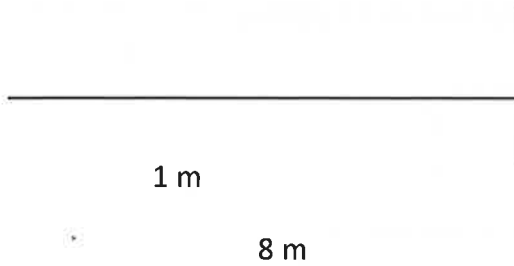
17	Draw bending moment diagram for a simply supported beam of length L with an anti-clockwise moment M applied at the mid-span.	Applying	2,4
18	Give the mathematical relationship between rate of loading, shear force and bending moment at a section in a beam.	Remembering	2,4
19	What do you mean by point of contraflexure?	Understanding	2,4

20	How many points of contraflexure you will have for simply supported beam overhanging at one end. Explain with a neat sketch.	Understanding	2,4
Part - B (Long Answer Questions)			
1	Derive the relation between rate of loading, shear force and bending moment for a beam carrying a uniformly distributed load of w per unit length over whole span.	Understanding	2,3,4
2	Derive the shear force and bending moment diagrams for a cantilever beam carrying a uniformly distributed load of w per unit run over half its span starting from the free-end.	Understanding	2,3,4
3	Draw the shear force diagrams for a cantilever beam of length 12 m carrying a uniformly distributed load of 12 kNm^{-1} over half its span starting from the free-end.	Applying	2,3,4
4	Draw the bending moment diagrams for a cantilever beam of length 12 m carrying a uniformly distributed load of 12 kNm^{-1} over half its span starting from the free-end.	Applying	2,3,4
5	Derive the shear force and bending moment diagrams for a cantilever beam carrying a uniformly varying load from zero at free end to w per unit length at the fixed end.	Applying	2,3,4
6	Draw the shear force and bending moment diagrams for a cantilever beam of length 4 m if two anti-clockwise moments of 15 kNm and 10 kNm are applied at the mid-span and the free end, respectively.	Applying	2,3,4
7	Draw the shear force and bending moment diagrams for a cantilever beam of length 7 m with a uniformly varying load from zero at fixed-end to 10 kN/m at 4m from the fixed end.	Applying	2,3,4
8	Draw the shear force and bending moment diagrams for a simply supported beam of length 12 m with an eccentric point load at a distance '3 m' from the	Applying	2,3,4

	left end and at a distance of '4m' from the right end.		
9	Derive the shear force and bending moment diagrams for a simply supported beam with an eccentric point load at a distance 'a' from left end and at a distance 'b' from right end.	Applying	2,3,4
10	Derive the shear force and bending moment diagrams for a simply supported beam carrying a uniformly distributed load of w per unit run over whole span.	Applying	2,3,4
11	Derive the shear force and bending moment diagrams for a simply supported beam carrying a uniformly varying load from zero at each end to w per unit length at the centre.	Applying	2,3,4
12	Derive the shear force and bending moment diagrams for a simply supported beam carrying a uniformly varying load from zero at one end to w per unit length at the other end.	Applying	2,3,4
13	Draw the shear force and bending moment diagrams for a simply supported beam of length 12 m with an eccentric point load of 20 kN at a distance '3 m' from the left end and of 20 kN at a distance of '3m' from the right end.	Applying	2,3,4
14	Draw the shear force and bending moment diagrams for a simply supported beam of length 12 m with an eccentric point load of 25 kN at a distance '3 m' from the left end and 20 kN at a distance of '4m' from the right end.	Applying	2,3,4
15	Draw the shear force and bending moment diagrams for a simply supported beam of length 10 m with a point load of 15 kN at the mid-span, and a uniformly varying load from zero at 5m from left end to 10 kN/m at the right end.	Applying	2,3,4
16	Draw the shear force and bending moment diagrams for the following beam 	Applying	2,3,4

17	Draw the shear force and bending moment diagrams for the following beam. 	Applying	2,3,4
18	Draw Shear Force and Bending Moment Diagram for a simply supported beam of length 20 m, with a triangular load on it full-span, the maximum value being 16 kN/m at the mid-point of the beam. 	Applying	2,3,4
19	Draw the shear force and bending moment diagrams for the following beam. 	Applying	2,3,4
	Draw the shear force and bending moment diagrams for the following beam. 		2,3,4

20.



Applying

Part – C (Problem Solving and Critical Thinking)

1	A cantilever beam of length 4m carries point loads of 1kN, 2kN and 3kN at 1, 2 and 4m from the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
2	A cantilever of length 4m carries a uniformly distributed load of 2kN/m run over the whole span and a point load of 2kN at a distance of 1m from the free end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
3	A cantilever of length 6m carries two point loads 2kN And 3kN at a distance of 1m and 6m from fixed end respectively. In addition to this the beam also carries a uniformly distributed load of 1kN/m over a length of 2m at a distance of 3m from the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
4	A cantilever of length 4m carries a uniformly distributed load of 3kN/m run over a length of 1m from the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
5	A cantilever of length 6m carries a gradually varying load, zero at the free end to 2kN/m at the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
6	A simply supported beam of length 8m carries point loads of 4kN and 6kN at a distance of 2m and 4m from the left end. Draw the S.F and B.M diagrams for	Analyze & evaluate	2,3,4

	the beam.		
7	A simply supported beam of length 6m is carrying a uniformly distributed load of 2kN/m from the right end. Draw the S.F and B.M diagrams for the beam.	Analyze & evaluate	2,3,4
8	A beam of length 10m is simply supported and carries point loads of 5kN each at a distance of 3m and 7m from the left end and also a uniformly distributed load of 1kN/m between the point loads. Draw the S.F and B.M diagrams for the beam.	Analyze & evaluate	2,3,4
9	A beam of length 6m is simply supported at its ends. It is loaded with gradually varying load of 750N/m from left support to 1500N/m to the right support. Construct the S.F and B.M diagrams and find the amount and position of maximum B.M over the beam.	Analyze & evaluate	2,3,4
10	A simply supported beam of length 8m rests on supports 6m apart, the right hand end is overhanging by 2m. The beam carries a uniformly distributed load of 1500N/m over the entire length. Draw S.F and B.M diagrams and find the point of contraflexure, if any.	Analyze & evaluate	2,3,4
11	A cantilever beam of length 8m carries point loads of 2kN, 4kN and 6kN at 2, 4 and 8m from the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
12	A cantilever of length 8m carries a uniformly distributed load of 4kN/m run over the whole span and a point load of 6 kN at a distance of 2m from the free end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
13	A cantilever of length 12 m carries two point loads 4 kN and 6 kN at a distance of 2m and 6m from fixed end respectively. In addition to this the beam also carries a uniformly distributed load of 2kN/m over a length of 4m at a distance	Analyze & evaluate	2,3,4

	of 6m from the fixed end. Draw the S.F and B.M diagrams for the cantilever.		
14	A cantilever of length 8m carries a uniformly distributed load of 4kN/m run over a length of 2m from the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
15	A cantilever of length 16m carries a gradually varying load, zero at the free end to 20 kN/m at the fixed end. Draw the S.F and B.M diagrams for the cantilever.	Analyze & evaluate	2,3,4
16	A simply supported beam of length 12 m carries point loads of 6 kN and 8 kN at a distance of 4m and 8m from the left end. Draw the S.F and B.M diagrams for the beam.	Analyze & evaluate	2,3,4
17	A simply supported beam of length 10 m is carrying a uniformly distributed load of 2kN/m for 4m from the right end. Draw the S.F and B.M diagrams for the beam.	Analyze & evaluate	2,3,4
18	A beam of length 20m is simply supported and carries point loads of 10 kN each at a distance of 6m and 14m from the left end and also a uniformly distributed load of 2 kN/m between the point loads. Draw the S.F and B.M diagrams for the beam.	Analyze & evaluate	2,3,4
19	A beam of length 12m is simply supported at its ends. It is loaded with gradually varying load of 1500N/m from left support to 3000N/m to the right support. Construct the S.F and B.M diagrams and find the amount and position of maximum B.M over the beam.	Analyze & evaluate	2,3,4
20	A simply supported beam of length 16m rests on supports 12m apart, the right hand end is overhanging by 4m. The beam carries a uniformly distributed load of 3000N/m over the entire length. Draw S.F and B.M diagrams and find the	Analyze & evaluate	2,3,4

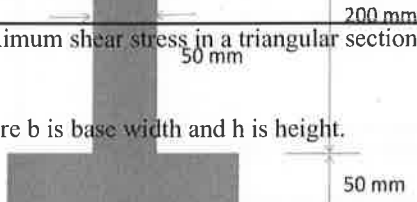
	point of contraflexure, if any.		
UNIT-III FLEXURAL STRESSES, SHEAR STRESSES			
Part - A (Short Answer Questions)			
1	Define bending stress in a beam with a diagram.	Understanding	5
2	Define pure bending and show an example by a figure.	Understanding	5
3	Define neutral axis and where is it located in a beam.	Understanding	5
4	What are the assumptions made in theory of simple bending?	Remembering	5
5	Write the bending equation, defining all the terms in the equation.	Remembering	5
6	Explain the terms: moment of resistance and section modulus	Remembering	5,6
7	Explain the role of section modulus in defining the strength of a section.	Understanding	5,6
8	Write the section modulus for a solid rectangular section.	Applying	5,6
9	Write the section modulus for a hollow rectangular section.	Applying	5,6
10	Write the section modulus for a solid circular section.	Applying	5,6
11	Of the following sections: rectangular, circular, triangular, I, T sections, which is most efficient for withstanding bending? Why?	Understanding	5,6
12	Under which conditions is the simple bending theory valid in practical applications?	Understanding	5
13	What do you mean by shear stress in beams?	Understanding	5
14	Write the expression for shear stress in a section of beam and explain the	Understanding	5

	terms.		
15	Draw the bending stress and shear stress profiles for a rectangular beam section.	Understanding	5
16	Draw the bending stress and shear stress profiles for a circular beam section.	Understanding	5
17	Draw the bending stress and shear stress profiles for a hollow rectangular beam section.	Understanding	5
18	Draw the bending stress and shear stress profiles for a hollow circular beam section.	Understanding	5
19	Explain the concept of complimentary shear in longitudinal section of a beam which is transversely loaded.	Understanding	5
20	Of the following sections: rectangular, circular, triangular, I, T sections, which is most efficient for withstanding shearing stresses in beams? Why?	Understanding	5

Part – B (Long Answer Questions)

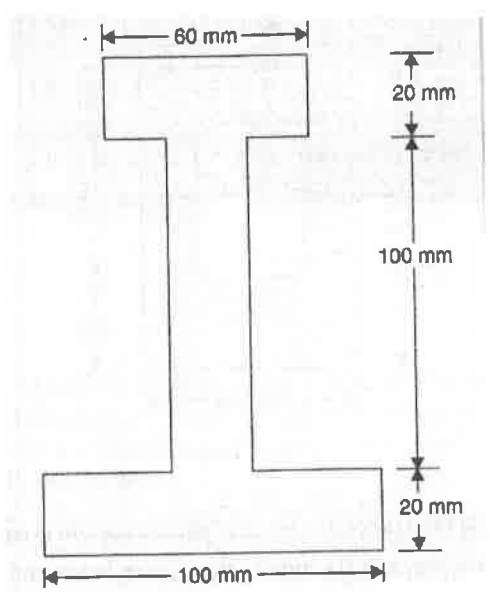
1	Derive the bending equation for a beam.	Understanding	5
2	For a given stress, compare the moments of resistance of a beam of a square section, when placed (i) with its two sides horizontal and (ii) with its diagonal horizontal. Which is more suitable?	Understanding	5,6
3	Three beams have the same length, the same allowable stress and the same bending moment. The cross-section of the beams are a square, a rectangle with depth twice the width and a circle. If all the three beams have the same flexural resistance capacity, then find the ratio of the weights of the beams. Which beam is most economical?	Understanding	5,6
4	A rectangular beam 60 mm wide and 150 mm deep is simply supported over a span of 6 m. If the beam is subjected to central point load of 12 kN, find the maximum bending stress induced in the beam section.	Applying	5,6
	A rectangular beam 300 mm deep is simply supported over a span of 4 m.		

5	What uniformly distributed load the beam may carry, if the bending stress is not to exceed 120 MPa. Take $I = 225 \times 10^6 \text{ mm}^4$.	Applying	5,6
6	A cantilever beam is rectangular in section having 80 mm width and 120 mm depth. If the cantilever is subjected to a point load of 6 kN at the free end and the bending stress is not to exceed 40 MPa, find the span of the cantilever beam.	Applying	5,6
7	A hollow square section with outer and inner dimensions of 50 mm and 40 mm respectively, is used as a cantilever of span 1 m. How much concentrated load can be applied at the free end, if the maximum bending stress is not exceed 35 MPa?	Applying	5,6
8	A cast iron water pipe of 500 mm inside diameter and 20 mm thick is supported over a span of 10 m. Find the maximum stress in the pipe metal, when the pipe is running full. Take density of cast iron as 70.6 kN/m^3 , and that of water as 9.8 kN/m^3 .	Applying	5,6
9	Two wooden planks 150 mm x 50 mm each are connected to form a T-section of a beam. If a moment of 6.4 kNm is applied around the horizontal neutral axis, find the bending stresses at both the extreme fibres of the cross-section.	Applying	5,6
10	Consider the following rolled steel beam of an unsymmetrical I-Section. If the maximum bending stress in the beam section is not to exceed 40 MPa, find the maximum moment which the beam can resist.	Applying	5,6
11	Prove that shear stress at any point in the cross-section of a beam which is subjected to a shear force F , is given by —	Applying	5,6

12	Show that for a rectangular section of the maximum shear stress is 1.5 times the average stress.	Applying	5,6
13	Prove that the shear stress distribution in a rectangular section of beam which is subjected to a shear force F is given by 	Applying	5,6
14	Prove that maximum shear stress in a circular section of beam is $\frac{4}{3}$ times the average shear stress.	Applying	5,6
15	Prove that the maximum shear stress in a triangular section of a beam is given by  where b is base width and h is height.	Applying	5,6
16	Show that the ratio of maximum shear stress to mean shear stress in a rectangular cross-section is equal to 1.50 when it is subjected to a transverse shear force F . Plot the variation of shear stress across the section.	Applying	5,6
17	Sketch the distribution of shear stress across the depth of the beams of the following cross-sections: (i) T-section; (ii) square section with diagonal horizontal.	Applying	5,6
18	A rectangular beam section 100 mm wide is subjected to a maximum shear force of 50 kN. Find the depth of the beam, if the maximum shear stress is 3 MPa.	Applying	5,6
19	A circular beam of 100 mm diameter is subjected to a shear force of 30 kN. Calculate the value of maximum shear stress and sketch the variation of shear stress along the depth of the beam.	Applying	5,6
20	An I section with rectangular ends, has the following dimensions: Flanges = 150 mm x 20 mm, Web = 300 mm x 10 mm. Find the maximum shearing stress developed in the beam for a shear force of 50 kN.	Applying	5,6

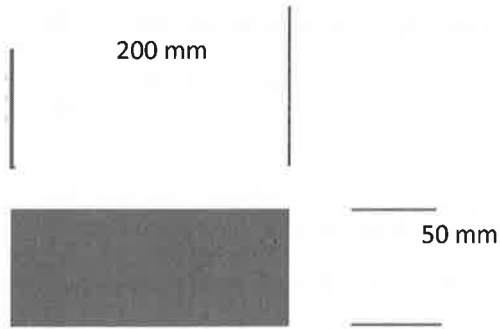
Part – C (Problem Solving and Critical Thinking)

1	A square beam 20mm x 20mm in section and 2m long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per meter length will break a cantilever of same material 40mm wide, 60mm deep and 3m long?	Analyze & evaluate	5,6
2	A rectangular beam is to be cut from a circular log of wood of diameter D. Find the ratio of the dimensions of the strongest section to resist bending stresses.	Analyze & evaluate	5,6
3	An I-section shown in figure is simply supported over a span of 12m. If the maximum permissible bending stress is 80N/mm^2, what concentrated load can be carried at a distance of 4m from one support?	Analyze & evaluate	5,6
4	Two circular beams where one is solid of diameter D and other is a hollow of outer dia. D_o and inner dia. D_i are of same length, same material and of same weight. Find the ratio of these circular beams.	Analyze & evaluate	5,6
	A cast iron beam is of T-section as shown in figure. The beam is simply supported on a span of 8m. The beam carries a uniformly distributed load of		

5	1.5kN/m length on the entire span. Determine the maximum tensile and maximum compressive stress.	Analyze & evaluate	5,6
6	A beam of I-section shown in figure is simply supported over a span of 4m. Determine the load that the beam can carry per meter length, if the allowable stress in the beam is 30.82 N/mm^2. 	Analyze & evaluate	5,6
7	A timber beam of rectangular section is simply supported at the ends and carries a point load at the centre of the beam. The maximum bending stress is 12 N/mm^2 and maximum shearing stress is 1 N/mm^2, find the ratio of span to depth.	Analyze & evaluate	5,6

8	A circular beam of 100 mm diameter is subjected to a shear force of 5kN. Calculate: (i) average shear stress, (ii) Maximum shear stress and (iii) shear stress at a distance of 40mm from NA.	Analyze & evaluate	5,6
9	A timber beam 100mm wide and 150mm deep supports a uniformly distributed load of intensity w kN/m length over a span of 2m. If the safe stresses are 28 N/mm ² in bending and 2 N/mm ² in shear, calculate the safe intensity of the load which can be supported by the beam.	Analyze & evaluate	5,6
10	An I-section has the following dimensions: Flange: 150mm x 20mm, Web: 30mm x 10mm The maximum shear stress developed in the beam is 16.8 N/m ² . Find the shear force to which the beam is subjected.	Analyze & evaluate	5,6
11	The maximum shear stress in a beam of circular section of diameter 150mm is 5.28 N/mm ² . Find the shear force to which the beam is subjected.	Analyze & evaluate	5,6
12	A rectangular beam 30mm x 20mm in section and 3m long is supported at the ends. The beam fails when a point load of 600 N is applied at the centre of the beam. What uniformly distributed load per meter length will break a cantilever of same material 40mm wide, 60mm deep and 3m long?	Analyze & evaluate	5,6
13	A steel beam of rectangular section is simply supported at the ends and carries a point load at the centre of the beam. The maximum bending stress is 40 N/mm ² and maximum shearing stress is 8 N/mm ² , find the ratio of span to depth.	Analyze & evaluate	5,6
14	An I-section beam 350 mm x 200 mm has a web thickness of 12.5 mm and a flange thickness of 25 mm. It carries a shearing force of 200 kN at a section. Sketch the shear stress distribution across the section.	Analyze & evaluate	5,6
A T-shaped cross-section of a beam as shown is subjected to a vertical shear force of 100 kN. Calculate the shear stress at important points and draw shear stress distribution diagram. Moment of inertia about the horizontal neutral axis is $113.4 \times 10^6 \text{ mm}^4$.			

15



Analyze &
evaluate

5,6

200 mm

50 mm

UNIT-IV

PRINCIPAL STRESSES AND STRAINS, THEORIES OF FAILURE

Part – A (Short Answer Questions)

1	Define principal planes and principal stresses	Understanding	7,8
2	Why is it important to determine principal stresses and planes?	Understanding	7,8
3	What are the methods used to determine the stresses on oblique section?	Remembering	7,8
4	Draw the representation of biaxial state of stress at a point in a material.	Understanding	7,8
5	Draw the representation of the state of pure shear stress at a point in a material.	Understanding	7,8
6	Explain the condition of plane stress.	Understanding	7,8
7	Write the expression for normal and tangential stresses on an inclined plane for a material element subjected to combined biaxial and shear stress.	Understanding	7,8
8	Give the expression for principal stresses for the case of combined bi-axial and	Understanding	7,8

	shear stress (plan stress condition).		
9	Give the expression for maximum shear stress for the case of combined bi-axial and shear stress (plan stress condition).	Understanding	7,8
10	Explain Mohr's circle of stresses using an example.	Understanding	7,8
11	List the various theories of failure of materials.	Remembering	9
12	State the Maximum Principal Stress Theory of Failure.	Remembering	9
13	State the Maximum Principal Strain Theory of Failure.	Remembering	9
14	State the Maximum Shear Stress Theory of Failure.	Remembering	9
15	State the Maximum Strain Energy Theory of Failure.	Remembering	9
16	State the Maximum Shear Strain Energy Theory of Failure.	Remembering	9
17	State the distortion energy theorem for failure.	Remembering	9
18	Which theory of failure is best suited for ductile materials? Why?	Understanding	9
19	Which theory of failure is best suited for brittle materials? Why?	Understanding	9
20	List the theories of material failure with their applicability to different materials	Understanding	9

Part – B (Long Answer Questions)

1	A rectangular bar is subjected to a direct stress (σ) in one plane only. Prove that the normal and shear stresses on an oblique plane are given by $\sigma_n = \sigma \cos^2 \theta$ $\tau = \sigma \sin \theta \cos \theta$ Where θ = angle made by oblique plane with the normal cross-section of bar σ_n = normal stress; τ = tangential or shear stress	Applying	7, 8
	A rectangular bar is subjected to two direct stresses σ_1 and σ_2 in two mutually		

2	perpendicular directions. Prove that the normal stress and shear stress on an oblique plane which is inclined at an angle θ with the axis of minor stress are given by _____ and _____	Applying	7, 8
3	Derive an expression for the stresses on an oblique plane of a rectangular body, when the body is subjected to a simple shear stress.	Applying	7, 8
4	A rectangular body is subjected to direct stresses in two mutually perpendicular directions accompanied by a shear stress. Prove that the normal stress and shear stress on an oblique plane inclined at angle θ with the plane of major direct stress are given by _____ and _____	Applying	7, 8
5	Derive an expression for the major and minor principal stresses on an oblique plane, when the body is subjected to direct stresses in two mutually perpendicular directions accompanied by a shear stress.	Applying	7, 8
6	Define and explain the theories of failure: (i) Maximum principal stress theory (ii) Maximum principal strain theory	Understanding	7, 8, 9
7	Define and explain the theories of failure: (i) Maximum shear stress theory (ii) Maximum shear strain energy theory	Understanding	7, 8, 9
8	A body is subjected to direct stresses in two mutually perpendicular principal tensile stresses accompanied by a simple shear stress. Draw the Mohr's circle of stresses and explain how you will obtain the principal stresses and strains.	Applying	7, 8
9	A body is subjected to direct stresses in two mutually perpendicular directions. How will you determine graphically the resultant stresses on an oblique plane when (i) the stresses are unequal and unlike; (ii) the stresses are unequal and like.	Applying	7, 8

10	In a two dimensional stress system, the direct stresses on two mutually perpendicular planes are 100 MN/mm^2 . These planes also carry a shear stress of 25 MN/mm^2 . If the factor of safety on elastic limit is 2.5, then find: (i) the value of stress when shear strain energy is minimum; (ii) elastic limit of material in simple tension.	Applying	7, 8
11	Determine the diameter of a bolt which is subjected to an axial pull of 18 kN together with a transverse shear force of 9 kN, when the elastic limit in tension is 350 N/mm^2 , factor of safety = 3 and $\mu = 0.3$ using (i) Maximum principal stress theory (ii) Maximum principal strain theory (iii) Maximum shear stress theory (iv) Maximum strain energy theorem (v) Maximum shear strain energy theory	Applying	7, 8, 9
12	A bolt is under an axial thrust of 10 kN together with a transverse shear force of 4 kN. Calculate the diameter of bolt according to (i) Maximum principal stress theory (ii) Maximum shear stress theory (iii) Maximum strain energy theorem Take elastic limit in simple tension = 225 N/mm^2 , factor of safety = 3, $\mu = 0.3$.	Applying	7, 8, 9

The principal stresses at a point in a elastic material are 25 N/mm^2 (tensile),

100 N/mm^2 (tensile) and 50 N/mm^2 (compressive). If the elastic limit in simple

tension is 220 N/mm^2 and $\mu = 0.3$, then determine whether the failure of

13	material will occur or not according to	Applying	7, 8, 9
	(i) Maximum principal stress theory		
	(ii) Maximum principal strain theory		
	(iii) Maximum shear stress theory		
	(iv) Maximum strain energy theorem		

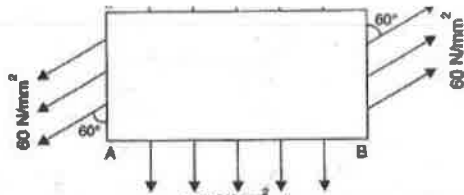
Maximum shear strain energy theory

The principal stresses at a point in a elastic material are 30 N/mm^2 (tensile), 120 N/mm^2 (tensile) and 50 N/mm^2 (compressive). If the elastic limit in simple tension is 250 N/mm^2 and $\mu_r = 0.3$, then determine whether the failure of		
14	material will occur or not according to (iii) Maximum principal stress theory (iv) Maximum principal strain theory (v) Maximum shear stress theory (vi) Maximum strain energy theorem (vii) Maximum shear strain energy theory The stresses at a point in a bar are 250 N/mm^2 (tensile) and 150 N/mm^2 (compressive). Determine the resultant stress in magnitude and	Applying 7, 8, 9
15	direction on a plane inclined at 30° to the axis of major stress. Also determine the maximum intensity of shear stress in the material at that point.	Applying 7, 8
The normal stress in two mutually perpendicular directions are 500 N/mm^2 and 320 N/mm^2 both tensile. The complimentary shear stress in these directions is		
16	of intensities 350 N/mm^2 . Find the normal and tangential stresses on the two planes which are equally inclined to the plane carrying the normal stress mentioned above.	Applying 7, 8
The axial stresses at a point in a bar are -100 N/mm^2 and 150 N/mm^2 , and the		
17	shear stress is 150 N/mm^2 . Determine the maximum intensity of shear stress in the material at that point.	Applying 7, 8
The normal strains in two mutually perpendicular directions are -2.3 and 5.0 and shear strain is 3.0 . Find the minimum and maximum values of the strains.		
18		Applying 7, 8
A bolt of 12 mm diameter is subjected to an axial pull of 10 kN and a shear force of 7.5 kN . Determine the factor of safety against failure based on various theories, if the yield strength of the material is 400 MPa , and Poisson's ratio is 0.3 .		
19		Applying 7, 8
A bolt of 18 mm diameter is subjected to an axial force of 25 kN . Determine		
20	the maximum shear force the bolt can sustain according to various theories, if	Applying 7, 8

the yield strength of the material is 340 MPa, and factor of safety is 1.8.

Part – C (Problem Solving and Critical Thinking)

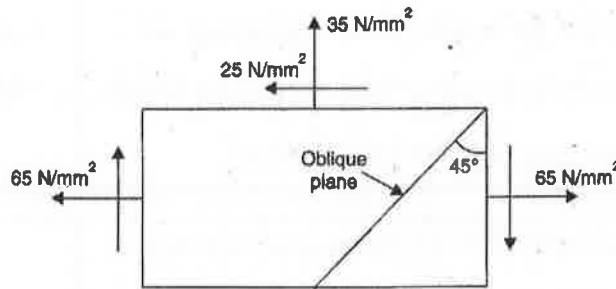
1	The stresses at a point in a bar are 200 N/mm^2 (tensile) and 100 N/mm^2 (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of major stress. Also determine the maximum intensity of shear stress in the material at that point.	Analyze & evaluate	7, 8
2	A point in a strained material is subjected to the stresses as shown in figure. Locate the principal planes, and evaluate the principal stresses.	Analyze & evaluate	7, 8
3	The normal stress in two mutually perpendicular directions are 600 N/mm^2 and 300 N/mm^2 both tensile. The complimentary shear stress in these directions is of intensities 450 N/mm^2 . Find the normal and tangential stresses on the two planes which are equally inclined to the plane carrying the normal stress mentioned above.	Analyze & evaluate	7, 8
4	The normal stress in two mutually perpendicular directions are 520 N/mm^2 and 360 N/mm^2 both tensile. The complimentary shear stress in these directions is of intensities 420 N/mm^2 . Find the normal and tangential stresses on the two planes which are equally inclined to the plane carrying the normal stress mentioned above.	Analyze & evaluate	7, 8
	A point in a strained material is subjected to stress shown in figure. Using		



Mohr's circle method, determine the normal and tangential stresses across the oblique plane. Check the answer analytically.

Apply &
evaluate

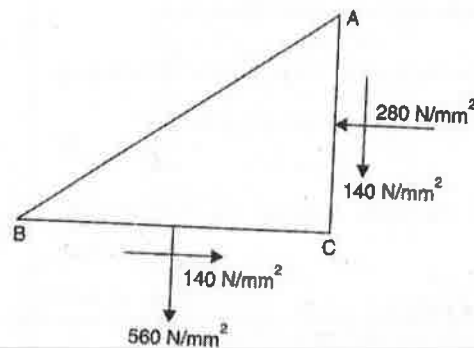
7, 8



At a point in a strained material, on plane BC there are normal and shear stresses of 560 N/mm^2 and 140 N/mm^2 respectively. On plane AC, perpendicular to plane BC, there are normal and shear stresses of 280 N/mm^2 and 140 N/mm^2 respectively as shown in the figure. Determine the following:
(i) principal stresses and location on which they act; (ii) maximum shear stress and the plane on which it acts.

Analyze &
evaluate

7, 8



7	The principal stresses at a point in a elastic material are 22 N/mm^2 (tensile), 110 N/mm^2 (tensile) and 55 N/mm^2 (compressive). If the elastic limit in simple tension is 220 N/mm^2 and $\mu = 0.3$, then determine whether the failure of material will occur or not according to	Analyze & evaluate	7, 8, 9
	(v) Maximum principal stress theory (vi) Maximum principal strain theory (viii) Maximum shear stress theory (ix) Maximum strain energy theorem (x) Maximum shear strain energy theory		
8	Determine the diameter of a bolt which is subjected to an axial pull of 12 kN together with a transverse shear force of 6 kN, when the elastic limit in tension is 300 N/mm^2, factor of safety = 3 and $\mu = 0.3$ using (vi) Maximum principal stress theory (vii) Maximum principal strain theory (viii) Maximum shear stress theory (ix) Maximum strain energy theorem (x) Maximum shear strain energy theory	Analyze & evaluate	7, 8, 9
9	A bolt is under an axial thrust of 7.2 kN together with a transverse shear force of 3.6 kN. Calculate the diameter of bolt according to (iv) Maximum principal stress theory (v) Maximum shear stress theory (vi) Maximum strain energy theorem Take elastic limit in simple tension = 202 N/mm^2, factor of safety = 3, $\mu = 0.3$.	Analyze & evaluate	7, 8, 9
10	In a two dimensional stress system, the direct stresses on two mutually perpendicular planes are 120 MN/mm^2. These planes also carry a shear stress of 40 MN/mm^2. If the factor of safety on elastic limit is 3, then find: (i) the value of stress when shear strain energy is minimum; (ii) elastic limit of material in simple tension.	Analyze & evaluate	7, 8, 9

Determine the maximum stresses in the body for the stress conditions: $\sigma_x = 80$

- 11 N/mm^2 , $\sigma_y = -100 \text{ N/mm}^2$ and $\tau_{xy} = -150 \text{ N/mm}^2$. The complimentary shear stress in these directions is of intensities 350 N/mm^2 . Indicate their planes, and the stresses on the planes inclined at 30° and -60° with the Y-axis.

Analyze &
evaluate

7, 8, 9

- * Develop Mohr's circle for the stress conditions $\sigma_x = +300 \text{ N/mm}^2$, $\sigma_y = -400 \text{ N/mm}^2$, $\sigma_z = +100 \text{ N/mm}^2$ and $\tau_{xy} = -250 \text{ N/mm}^2$. Determine the magnitudes and the planes of principal stresses and maximum shear stresses (DSPR).

Analyze &
evaluate

7, 8, 9

- * Stresses at a point in a body are estimated as $\sigma_0 = +150 \text{ N/mm}^2$, $\sigma_{45} = +250 \text{ N/mm}^2$ and $\sigma_{60} = +100 \text{ N/mm}^2$. Determine the principal stresses and maximum shear stress along with their planes. The subscripts indicate the orientation of the stress direction with the x-axis.

Analyze &
evaluate

7, 8, 9

- Determine the planes of principal strains and maximum shear strains, if the strains measured in a body are $\epsilon_x = 1.25$, $\epsilon_y = 3.25$ and $\epsilon_{60} = 3$, the values being in the units of microstrains.

Analyze &
evaluate

7, 8, 9

- Determine the planes of principal strains and maximum shear strains, if the strains measured in a body are $\epsilon_x = 2.85$, $\epsilon_{60} = -7$ and $\epsilon_{120} = -5$, the values being in the units of microstrains.

Analyze &
evaluate

7, 8, 9

UNIT-V

DEFLECTION OF BEAMS

Part - A (Short Answer Questions)

1	Define deflection and slope of a beam.	Remembering	10
2	Write the differential equation for the beam	Remembering	10
3	List the different methods for finding slope and deflection of a beam.	Remembering	10
4	Explain the concept of double-integration method to obtain the deflections of a beam	Understanding	10

5	Give the relation between the load, shear force and bending moment at a section of a beam.	Remembering	10
6	What is Macaulay's method? How is it different from the general double integration method?	Understanding	10
7	What is meant by flexural rigidity? Give its expression.	Remembering	10
8	Give the slope and deflection of a cantilever beam, with flexural rigidity EI , and length L , carrying a point load W at its free end?	Remembering	10
9	Give the slope and deflection of a simply supported beam, with flexural rigidity EI , and length L , carrying a point load W at its mid-span?	Remembering	10
10	State and explain the first theorem of Mohr.	Understanding	10
11	State and explain the second theorem of Mohr.	Understanding	10
12	How is moment area method used to calculate deflection of the free end of a cantilever? (Note: Just explain the concept and procedure in brief. Do not do calculations)	Understanding	10
13	Explain the concept of conjugate beam method.	Understanding	10
14	What is the advantage of the conjugate beam method over other methods?	Understanding	10
15	Write the boundary support conditions for slope and deflections of a cantilever, and write the same for its conjugate beam.	Understanding	10
16	Write the boundary support conditions for slope and deflections of a simply supported beam, and write the same for its conjugate beam.	Understanding	10
17	Draw the conjugate beam for a propped cantilever beam (one end fixed and other end on roller support).	Understanding	10
18	Draw the conjugate beam for a simply supported beam with an overhang on other end.	Understanding	10
19	How will you use conjugate beam method for finding slope and deflection at any section of a given beam?	Understanding	10

20	What is the relation between an actual beam and the corresponding conjugate beam for different end conditions?	Understanding	10
Part - B (Long Answer Questions)			
1	Derive an expression for slope and deflection of a beam subjected to uniform bending moment.	Applying	10
2	Prove that the relation $\frac{1}{EI} \int M dx = \theta$ where M is the bending-moment and E is modulus of elasticity and I is moment of inertia of the beam section.	Applying	10
3	Prove that the deflection at centre of a simply supported beam, carrying a point load at centre, is given by $\frac{wl^3}{48EI}$	Applying	10
4	Derive the slope at supports and deflection at centre for a simply supported beam carrying uniformly distributed load of w per unit length over the entire span.	Applying	10
5	Use Moment-Area method to find the slope and deflection of a simply supported beam carrying a point load at the centre.	Applying	10
6	Use Moment-Area method to find the slope and deflection of a simply supported beam carrying a uniformly distributed load over the entire span.	Applying	10
7	Derive slope and deflection of a cantilever carrying uniformly distributed load over whole length using Macaulay's method.	Applying	10
8	Derive slope and deflection of a cantilever carrying uniformly distributed load over a length 'a' from the fixed end by double integration method.	Applying	10
9	Derive slope and deflection of a cantilever carrying uniformly distributed load over a length 'a' from the fixed end by Moment-Area method.	Applying	10
10	Derive slope and deflection relations for a cantilever carrying a gradually	Applying	10

	varying load from zero at the free end to w per metre run at the fixed end.		
11	Find the slope and deflection of a simply supported beam carrying a point load centre, using conjugate beam method.	Applying	10
12	A cantilever carries a point load at the free end. Determine the deflection at free end using conjugate beam method.	Applying	10
13	Determine the deflection of the beam at the point of application of the 300 Nm couple as shown in the figure. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 200 \text{ cm}^4$.	Applying	10
	A simply supported beam shown in the Figure carries a uniformly distributed load of intensity w symmetrically distributed over part of its length. Determine the maximum deflection and check your results by assuming $a = 0$ and compare, when same beam is with u.d.l. on the entire span.		
14		Applying	10
15	A steel Cantilever of 2.5m effective length carries a load of 25 kN at its free end. If the deflection at the free end is not to exceed 0.5 cm, what must be the I value of the section of the cantilever? Use Moment Area Method. Take $E = 210 \text{ GPa}$.	Applying	10

A simply supported beam 5 m long carries concentrated loads of 10 kN each at a distance 1m from the ends. Calculate:

- | | | | |
|----|---|----------|----|
| 16 | (a) Maximum slope and deflection for the beam, and
(b) Slope and deflection under each load. | Applying | 10 |
|----|---|----------|----|

Take: $EI = 1.2 \times 10^4 \text{ kN.m}^2$.

A cantilever of length L is loaded with uniformly varying load of intensity zero

- | | | | |
|----|---|----------|----|
| 17 | at the free end and w/unit length at the fixed end. Derive an expression for the deflection at any point. Find also the slope and deflection of the free end. | Applying | 10 |
|----|---|----------|----|

A beam 6 m long, simply supported at its ends, is carrying a point load of 50 kN at its centre. The moment of inertia of the beam is given as equal to $78 \times$

- | | | | |
|----|--|----------|----|
| 18 | 10^6 mm^4 . If E for the material of the beam = $2.1 \times 10^5 \text{ N/mm}^2$, Calculate:
(a) deflection at the centre of the beam, and
(b) slope at the supports. | Applying | 10 |
|----|--|----------|----|

A simply supported beam of circular cross-section is 5 m long and is of 150 mm diameter. What will be the maximum value of the central load if the deflection of the beam does not exceed 12.45 mm. Also calculate the slope at the supports. Take $E = 2 \times 10^8 \text{ kN/m}^2$.

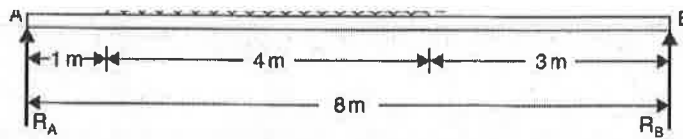
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|----|--|----------|----|
| 19 | | Applying | 10 |
| 20 | A cantilever of 4m span length carries a load 40 KN at its free end. If the deflection at the free end is not to exceed 8mm, what must be the moment of inertia of the Cantilever section? | Applying | 10 |

Part – C (Problem Solving and Critical Thinking)

A beam of length 6m is simply supported at its ends and carries two point loads of 48kN and 40kN at a distance of 1m and 3m respectively from the left

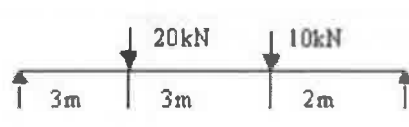
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|---|---|---------------------|----|
| 1 | support. Find: (i) deflection under each load, (ii) maximum deflection and (iii) the point at which maximum deflection occurs. Given $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 85 \times 10^6 \text{ mm}^4$. | Apply &
evaluate | 10 |
|---|---|---------------------|----|

2	A beam of length 8m is simply supported at its ends. It carries a uniformly distributed load of 40kN/m as shown in figure below. Determine the deflection of the beam at its midpoint and also the position of maximum deflection and maximum deflection. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 4.3 \times 10^8 \text{ mm}^4$.	Apply & evaluate	10
3	A beam ABC of length 9m has one support to the left end and the other support at a distance of 6m from the left end. The beam carries a point load of 1kN at the right end and also carries a uniformly distributed load of 4kN/m over a length of 3m as shown in the figure. Determine slope and deflection at point C. $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 5 \times 10^8 \text{ mm}^4$.	Apply & evaluate	10
4	A beam of 4.8m and of uniform rectangular section is simply supported at its ends. It carries a uniformly distributed load of 9.375kN/m run over the entire length. Calculate the width and depth of the beam if permissible bending stress is 7 N/mm^2 and maximum deflection is not to exceed 0.95cm. Take E for beam material = $1.05 \times 10^4 \text{ N/mm}^2$	Apply & evaluate	10
	A beam ABC of length 9m has one support to the left end and the other support at a distance of 6m from the left end. The beam carries a point load of 1kN at the right end and also carries a uniformly distributed load of 4kN/m over a length of 3m as shown in the figure. Determine slope and deflection at		



5	point C. $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 5 \times 10^8 \text{ mm}^4$. Use moment-area method.	Apply & evaluate	10
6	Determine the deflection at the free end of a cantilever which is 2m long and carries a point load of 9kN at the free end and a uniformly distributed load of 8kN/m over a length of 1m from the fixed end. Take $E = 2.2 \times 10^5 \text{ N/mm}^2$ and $I = 2.25 \times 10^7 \text{ mm}^4$.	Apply & evaluate	10
7	A cantilever of length 2m carries a uniformly varying load of zero intensity at free end and 45kN/m at the fixed end. If $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 10^8 \text{ mm}^4$, find the slope and deflection of the free end.	Apply & evaluate	10
8	A cantilever of length 2m carries a point load of 3kN at the free end and another load of 30kN at its centre. If $EI = 10^{13} \text{ N/mm}^2$ for the cantilever, then determine by moment area method, the slope and deflection at the free end of cantilever.	Apply & evaluate	10
9	A beam of length 6m is simply supported at its ends and carries two point loads of 48kN and 40kN at a distance of 1m and 3m respectively from the left support. Find the deflection under each load. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 85 \times 10^6 \text{ mm}^4$. Use conjugate beam method.	Apply & evaluate	10

10	A cantilever of length 3m is carrying a point load of 50kN at a distance of 2m from the fixed end. If $I = 10^8 \text{ mm}^4$ and $E = 2 \times 10^5 \text{ N/mm}^2$, find slope and deflection at free end using conjugate beam method.	Apply & evaluate	10
11	A steel girder of uniform section, 14 meters long, is simply supported at its ends. It carries concentrated loads of 120 kN and 80 kN at two points 3 meters and 4.5 meters from the two ends respectively. (a) Calculate the deflection of the girder at the two points under the two loads: (b) The maximum deflection. Use Macaulay's Method. Take: $I = 16 \times 10^4 \text{ m}^4$, and $E = 210 \times 10^6 \text{ kN/m}^2$.		10
12	A horizontal beam of uniform section and length L rests on supports at its ends. It carries a U.D.L. w per unit length which extends over a length 'a' from the right hand support. Determine the value of 'a' in order that the maximum deflection may occur at the left hand end of the load, and if the maximum deflection is $wl^4/k EI$ determine the value of k.	Analyze & evaluate	10
13	Show that the central deflection in a symmetrical double over hanging beam of span L and over hangs "a" with concentrated loads W at free ends is $Wal^2/8EI$.	Analyze & evaluate	10
14	A horizontal beam of uniform section is pinned at its ends which are at the same level and is loaded at the left hand pin with an anticlockwise moment M and at the right hand pin with a clockwise moment 2m both in the same vertical plane. The length between the pins is L. Find the angles of slope at each end and the deflection of the midpoint of the span in terms of M, L, E and I.	Analyze & evaluate	10
	Determine the maximum deflection and the slope of the beam as shown in the		

15	figure using any one of the following methods: (a) Macaulay's method (b) moment-Area method (c) Conjugate beam method 	Analyze & evaluate	10
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18. MID WISE QUESTION PAPERS

Nawab Shah Alam Khan College of Engineering & Technology

New Malakpet, Hyderabad-500024

2nd Year 1st SEMESTER BTECH MID-I EAMINATIONS SEP 2018

BRANCH: CIVIL

DATE:

SUBJECT: STRENGTH OF MATERIAL - I
60mins

TIME:

Answer any two of the following questions.

2X5=10

1. Derive stress and strain diagram for mild steel.
2. A) What do you understand by Yielding, Strain hardening, Neck formation and Permanent set?
B) Find the minimum diameter of steel wire with which a load of 4000N can be raised so that the tensile stress in the wire may not exceed 130 N/mm². Calculate the extension of wire if it is 3m long. Take E= 200 GPa
3. A) If a tensile load of 50 KN is applied suddenly to a circular bar of 4cm diameter and 5m long, then determine.
 - i) Maximum instantaneous stress induced.
 - ii) Instantaneous elongation in rod.
 - iii) Strain energy absorbed in the rod.B) Derive Strain Energy.
4. Draw shear force and bending moment diagram of length 8m carrying UDL of 12KN for a distance of 4m from the left end and also calculate the maximum bending moment of the section.

Nawab Shah Alam Khan College of Engineering & Technology

New Malakpet, Hyderabad-500024

2nd Year 1st SEMESTER BTECH MID-I EAMINATIONS SEP 2018

BRANCH: CIVIL

DATE:

SUBJECT: STRENGTH OF MATERIAL - I
60mins

TIME:

Answer any two of the following questions.

2X5=10

1. Derive the equation $\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$
2. Calculate the ratio of maximum to mean shear stress in an I- beam 200 mm wide and 350 mm deep, having the flanges 25 mm thick and web 12.5 mm thick. Also find the percentage of the total shearing force carried by the web.
3. A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find
 - (i) Deflection under each load,
 - (ii) Maximum deflection, and
 - (iii) The point at which maximum deflection occurs.Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 85 \times 10^6 \text{ mm}^4$.
4. (a) A rectangular block is subjected to a tensile stress of 100 N/mm^2 on one plane and a tensile stress of 50 N/mm^2 on a perpendicular plane, along with shear stresses of 60 N/mm^2 on the same planes. Determine (i) direction of principal planes, (ii) magnitude of principal stress, (iii) maximum shear stress.
(b) The stresses at a point in a bar are 180 N/mm^2 (tensile) and 90 N/mm^2 (compressive). Determine the magnitude and direction of resultant stress on a plane inclined at 45° to the axis of major stress. Determine also the magnitude of maximum shear stress.

Nawab Shah Alam Khan College of Engineering & Technology

New Malakpet, Hyderabad-500024

2nd Year 1st SEMESTER BTECH MID-I EAMINATIONS SEP 2019

BRANCH: CIVIL

DATE:

SUBJECT: STRENGTH OF MATERIAL - I

TIME:

60mins

I Answer any two of the following

2x5=10

Q.no	Questions	Bloom's Level
1	Derive stress and strain diagram for mild steel.	L4
2	A) What do you understand by Yielding, Strain hardening, Neck formation and Permanent set?	L1
	B) Find the minimum diameter of steel wire with which a load of 4000n can be raised so that the tensile stress in the wire may not exceed 130 N/mm ² . Calculate the extension of wire if it is 3m long. Take E= 200 GPA.	L5
3	A) If a tensile load of 50 KN is applied suddenly to a circular bar of 4cm diameter and 5m long, then determine. i) Maximum instantaneous stress induced. iv) Instantaneous elongation in rod. v) Strain energy absorbed in the rod.	L5
	B)Derive Strain Energy.	L4
4	Draw shear force and bending moment diagram of length 8m carrying UDL of 12KN for a distance of 4m from the left end and also calculate the maximum bending moment of the section.	L5

Nawab Shah Alam Khan College of Engineering & Technology

New Malakpet, Hyderabad-500024

2nd Year 1st SEMESTER BTECH MID-II EAMINATIONS NOV 2019

BRANCH: CIVIL

DATE:

SUBJECT: STRENGTH OF MATERIAL - I
60mins

TIME:

I Answer any two of the following
2x5=10

Q.no	Questions	Bloom's Level
1	a) Derive an expression of shear stress produced in circular shaft subjected to torsion. b) Two shafts of same material and same length are subjected to same torque, if the first shaft is of solid circular section and second is a hollow circular section. The internal diameter is $\frac{2}{3}$ of outside diameter. Shear stress developed in each shaft is same, compare the weight of shaft.	L3
2	A leaf spring a central load of 3000 Newton the leaf spring is to be made of steel plates 5 cm wide and 6 mm thick. If the bending stress is limited to 150 N/mm². Determine A) Length of the spring B) Deflection at the centre of spring Take $E = 2 \times 10^5$ N/mm²	L1
	a) A mild steel tube 4 m long, 30 mm internal diameter and 4 mm thick is used as strut with both ends hinged. Find the crippling load.	

3	Take $E = 2.1 \times 10^5 \text{ N/mm}^2$. b) A hollow alloy tube 5 meter long with external and internal diameter equal to 40 mm and 25 mm respectively was found to be extended by 6.4 mm under a tensile load of 60KN. Find the buckling load for the tube when used as column with both ends pinned. Also find safe compressive load for the tube which factor of safety of 4.	L5
4	Steel strut, 1 m long, is 30 mm in diameter. It is subjected to an axial thrust of 18 KN. In addition, a lateral load W acts at the centre of the strut. If the strut fails at the maximum stress of 350 MN/m^2, determine the magnitude of W.	L1

**NAWAB SHAH ALAM KHAN COLLEGE OF ENGINEERING AND
TECHNOLOGY**

New Malakpet, Hyderabad-500024

B.E III SEMESTER CIE-II MARCH- 2021

**BRANCH: CIVIL ENGINEERING
2021**

DATE:03-03-

SUBJECT: SOLID MECHANICS

TIME:

PART-A

I Answer all the questions.

6X1=6

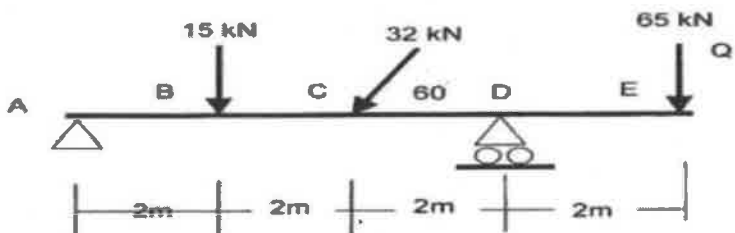
Q.No	Question	Bloom's Level
1	A cantilever span of 3m supports a uniformly varying load of 12KN/m at free end to 4KN/m at the fixed end. Sketch the BMD.	L1
2	State assumptions made in theory of simple bending.	L2
3	Write the formula for variation of shear stress 'q' across a section.	L2

PART-B

II Answer any two questions.

2X7=14

Q.No	Question	Bloom's Level
4	For a T section with dimensions flange width 100mm, depth=200mm, and uniform thickness 40mm. obtain the shear stress distribution and calculate maximum and average shear stresses if is subjected to a shear force = 100KN.	L1
5	Sketch the SFD and BMD for the overhanging simply supported beam loaded as shown in figure-1. Also find the point of contra flexure.	L1

	 FIG-I	
6	A rectangular beam 150mm wide and 300mm deep is simply supported over a span of 4m and carries a central point load 'W'. determine the magnitude of load 'W' if the permissible bending stress is 9 N/mm².	L5

22. References

1. Mechanics of Materials by B. C. Punmia, A. K. Jain and A. K. Jain, Laxmi Publications (P) Ltd, New Delhi.
2. Strength of Materials – A Practical Approach Vol.1 by D. S. Prakash Rao, Universities Press, Hyderabad, India.
3. Mechanics of materials by F. Beer, E. R. Johnston and J. Dewolf (Indian Edition – SI units), Tata McGraw Hill Publishing Co. Ltd., New Delhi
4. D.S. Prakash Rao, Strength of Materials- A Practical Approach, Universities Press, 1999.
5. R.K. Rajput, A Textbook of Strength of Materials, S. Chand Publications, 2007.
6. R. Subramanian, Strength of Materials, Oxford University Press, New Delhi 2005.
7. S. S. Bhavikatti, Strength of materials, Vikas Publishing House, 2002.
8. Ferdinand P Beer, Johnston and De Wolf., Mechanics of Materials, Tata McGraw-Hill, 2004

NAWAB SHAH ALAM KHAN COLLEGE OF ENGINEERING AND TECHNOLOGY						
DEPARTMENT OF CIVIL ENGINEERING						
CO Feedback form						
Academic Year 2020-2021						
Course Name with Code	SOM-1 & C213					
Class	BE Civil - III Semester					
Faculty Name	MOHD YOUSUF AHMED					
CO Attainment	Internal Attainment	External Attainment	DIRECT ATTAINMENT LEVEL	Indirect Attainment	Overall Attainment	COPO MAPPING
CO 1	3	3	3.00	2.29	3	1.57
CO 2	3	3	3.00	2.39	3	1.78
CO 3	3	3	3.00	2.29	3	1.57
CO 4	3	3	3.00	2.40	3	1.80
Overall Course Attainment			3.00	2.34	3.00	1.68
Set Target for the course						
Course Attainment						
Status(Yes/No)						YES

Percentage of students attained CO	CO attainment rubric
$\%CO \geq 80$	3
$65 \leq \%CO < 80$	2
$\%CO < 65$	1

H.O.D

NAWAB SHAH ALAM KHAN COLLEGE OF ENGINEERING AND TECHNOLOGY, OSMANIA UNIVERSITY, Hyderabad
DEPARTMENT OF CIVIL ENGINEERING

B.E. II YEAR, I/II SEM - ATTAINMENT CALCULATIONS - Academic Year: 2020-21

Subject: SOLID MECHANICS

Subject Code: EPC2210E

Faculty: MOHD YOUSUF AHMED

S.No.	Hall Ticket No.	CIE - 1											CIE - 2											CIE		SEE
		ASG-1 (5M)	ASG-2 (5 M)	Part-1 Q1-abcd (6 M)		Q 2 (7 M)	Q 3 (7 M)	BEST OF Q2&Q3	Q 4 (7 M)	Q 5 (7 M)	BEST OF Q4&Q5	CIE-1 TOTAL (30 M)	ASG-1 (5M)	ASG-2 (5 M)	Part-1 Q1abcd (6 M)		Q 2 (7 M)	Q 3 (7 M)	BEST OF Q2&Q3	Q 4 (7 M)	Q 5 (7 M)	BEST OF Q4&Q5	CIE-2 TOTAL (30 M)	Average CIE	TOTAL Marks	
		C01	C02	C01	C02	C01	C01	C01	C02	C02	C02		C03	C04	C03	C04	C03	C03	C03	C04	C04	C04		(30 M)	(100 M)	
1	161019732001	4	4	2	2	7		7	3		3	22	4	4	2	2	7		7		3	1	15	19	39	21
2	161019732002	3	4	3	3	4		4		7	7	24	3	4	3	3	4		4	7		7	24	21	49	29
3	161019732003	4	5	2	3		6	6	6		6	26	4	5	2	3		6	6		6	0	20	28	49	23
4	161019732004	4	4	2	2	6		6	7		7	25	4	4	2	2	6		6		7	7	25	22	39	18
5	161019732005	4	5	2	3		5	5			0	19	4	5	2	3		5	5			5	24	28	69	43
6	161019732006	4	4	2	2	4		4		3	3	19	4	4	2	2	4		4	3		5	21	24	39	17
7	161019732007	5	5	2	2		5	5	7		7	26	5	5	2	2		5	5		7	3	17	29	69	41
8	161019732008	4	4	2	3			0	5		5	18	4	4	2	3		0			5	7	26	29	49	22
9	161019732009	4	4	2	1	7		7			0	18	4	4	2	1	7		7			7	25	21	39	20
10	161019732010	4	4	2	2	4		4	3		3	19	4	4	2	2	4		4		3	0	16	22	39	18
11	161019732011	4	4	2	3		6	6		5	5	24	4	4	2	3		6	6	5		6	25	29	39	12
12	161019732013	4	4	2	2	3		3	7		7	22	4	4	2	2	3		3		7	4	18	20	49	31
13	161019732014	4	4	2	2	7		7		3	3	22	4	4	2	2	7		7	3		1	17	23	39	17
14	161019732016	4	4	2	2		4	4		3	3	19	4	4	2	2		4	4	3		0	16	22	39	21
15	161019732017	4	4	2	2	4		4		3	3	19	4	4	2	2	4		4	3		0	16	24	49	29
16	161019732018	4	4	2	2	7		7	4		4	23	4	4	2	2	7		7		4	6	25	19	39	23
17	161019732019	4	4	2	2		4	4	6		6	22	4	4	2	2		4	4		6	6	22	21	49	18
18	161019732020	5	6	2	2		7	7	1		1	23	5	6	2	2		7	7		1	2	24	28	59	43
19	161019732021	4	5	3	3	7		7			0	22	4	5	3	3	7		7			3	25	22	49	17
20	161019732022	4	4	2	2	2		2	2		2	16	4	4	2	2	2		2		2	3	17	28	39	41
21	161019732023	4	3	3	2		6	6		7	7	25	4	3	3	2		6	6	7		7	25	24	49	22
22	161019732024	4	4	2	2	4		4			0	16	4	4	2	2	4		4			7	25	29	49	20
23	161019732025	4	4	2	2		5	5	5		5	22	4	4	2	2		5	5		5	0	17	29	39	18
24	161019732026	4	5	3	3		6	6			0	21	4	5	3	3		6	6			6	27	21	49	12
25	161019732027	4	4	2	2		3	3			0	15	4	4	2	2		3	3			6	23	22	39	31
26	161019732028	4	4	2	2	4		4		5	5	21	4	4	2	2	4		4	5		5	24	29	49	17
27	161019732029	4	4	2	2	1		1	7		7	20	4	4	2	2	1		1		7	7	25	20	49	21
28	161019732030	4	4	3	3		6	6	5		5	25	4	4	3	3		6	6		5	0	20	23	59	29
29	161019732031	4	4	2	3		6	6	7		7	26	4	4	2	3		6	6		7	0	15	22	69	23
30	161019732032	4	4	2	3	7		7		5	5	25	4	4	2	3	7		7	5		3	16	24	49	18
31	161019732033	4	4	2	2	6		6	4		4	24	4	4	2	2	6		6		4	7	25	19	39	43
32	161019732034	4	4	2	2	2		2	5		5	19	4	4	2	2	2		2		5	7	21	21	59	17
33	161019732036	4	4	2	1		3	3	5		5	19	4	4	2	1		3	3		5	2	16	28	49	41
34	161019732038	4	4	2	1		5	5		6	6	22	4	4	2	1		5	5	6		6	21	22	69	22
35	161019732039	4	5	2	1		0			7	7	19	4	5	2	1		0	7			7	25	28	59	20
36	161019732040	4	5	3	2		7	7	3		3	24	4	5	3	2		7	7		3	0	17	24	59	18
37	161019732041	4	5	3	3	6		6	4		4	25	4	5	3	3	6		6		4	2	23	29	69	12
38	161019732042	4	5	3	3	6		6	3		3	24	4	5	3	3	6		6		3	1	22	29	49	31
39	161019732043	5	5	3	3		7	7			0	23	5	5	3	3		7	7			1	24	21	69	17
40	161019732044	5	5	3	2		4	4		6	6	25	5	5	3	2		4	4	6		0	19	22	59	21
41	161019732045	4	4	2	1		7	7		4	4	22	4	4	2	1		7	7	4		7	25	29	39	29
42	161019732046	4	4	2	3	2		2			0	15	4	4	2	3	2		2			7	27	20	59	23
43	161019732047	5	5	3	3		7	1	4		4	21	5	5	3	3		7	7		4	0	23	23	49	18
44	161019732050	5	5	3	2		5	5		5	5	25	5	5	3	2		5	5	5		5	27	22	59	43
45	161019732051	4	5	3	3	5		5	2		2	22	4	5	3	3	5		5		2	0	16	24	59	17
46	161019732052	4	5	3	3		6	6	4		4	25	4	5	3	3		6	6		4	3	24	19	39	41
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48	161019732055	4	5	3	3		6	6	2		2	23	4	5	3	3		6	6		2	0	15	28	69	20
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51	161019732058	4	3	3	1		3	3	7		7	21	4	3	3	1		3	3		7	4	18	24	39	31
52	161019732059	4	5	1	3	4		4			0	17	4	5	1	3	4		4			6	25	29	49	17
53	161019732061	4	5	3	3	4		4		6	6	25	4	5	3	3	4		4	6		6	27	29	69	21
54	161019732062	5	5	3	3	5		5	4		4	25	5	5	3	3	5		5		4	4	27	21	59	29
55	161019732063	4	5	3	3		4	4		7	7	26	4	5	3	3		4	4	7		7	29	22	79	23
56	161019732064	4	5	3	3		0	6			6	21	4	5												

85	161019732097	4	5	3	3		6	4	4	2		2	21	4	5	3	3		4	4		2	6	25	22	49	41
86	161019732098	4	5	3	3				6			0	21	4	5	3	3	6		6			5	27	29	49	22
87	161019732099	4	5	2	2			4	4	4		4	21	4	5	2	2		4	4		4	7	25	20	49	20
88	161019732301	4	5	3	3			5	5		7	7	27	4	5	3	3		5	5	7		7	29	23	69	18
89	161019732302	4	5	3	3			5	5		6	6	26	4	5	3	3		5	5	6		6	27	22	49	12
90	161019732303	4	5	2	3		6		6	6		6	26	4	5	2	3	6		6		6	26	24	69	31	
91	161019732304	4	5	2	3		2		2	6		6	22	4	5	2	3	2		2		6	6	27	19	69	17
92	161019732305	4	5	2	3			3	3		6	6	23	4	5	2	3		3	3	6		6	26	21	59	21
93	161019732306	4	5	2	3		1		1			0	15	4	5	2	3	1		1			0	15	28	69	29
94	161019732307	4	5	2	3			6	6	3		3	23	4	5	2	3		6	6		3	3	17	22	39	23
95	161019732308	4	5	2	3			7	7		6	6	27	4	5	2	3		7	7	6		6	26	28	79	18
96	161019732309	5	5	2	3			7	7	7		7	29	5	5	2	3		7	7		7	0	17	24	89	43
97	161019732310	4	5	2	3		5		5		6	6	25	4	5	2	3	5		5	6		6	25	29	59	17
98	161019732311	5	5	2	3			7	7	5		6	28	5	5	2	3		7	7		5	5	25	29	79	41
99	161019732312	4	4	2	3		6		6			7	26	4	4	2	3	6		6			0	16	21	39	22
100	161019732313	4	4	3	3		5		5	5		5	24	4	4	3	3	5		5		5	7	28	22	59	20
101	161019732314	4	5	2	3			6	6		7	7	27	4	5	2	3		6	6	7		5	19	29	79	18
102	161019732315	4	5	2	3			7	7		4	4	25	4	5	2	3		7	7	4		2	23	20	59	12
103	161019732316	4	4	2	3		4		4	2		2	19	4	4	2	3	4		4		2	1	18	23	39	31
104	161019732317	4	4	2	3		6		6		7	7	26	4	4	2	3	6		6	7		7	23	22	49	17
105	161019732318	4	4	2	3			2	2		5	5	20	4	4	2	3		2	2	5		5	23	24	59	21
106	161019732319	4	5	2	2			7	7	7		7	27	4	5	2	2		7	7		7	5	24	19	69	29
107	161019732320	4	5	2	2			5	5			0	18	4	5	2	2		5	5			5	24	21	49	23
108	161019732321	4	5	2	2				0	3		3	16	4	5	2	2			0		3	5	25	28	59	18
109	161019732322	4	5	2	3			5	5		5	5	24	4	5	2	3		5	5	5		7	27	22	59	43
110	161019732323	4	5	2	3			7	7		6	6	27	4	5	2	3	7		7	6		5	25	28	69	17
111	161019732324	4	5	2	2		5		5		7	7	25	4	5	2	2	5		5	7		2	17	24	49	41
112	161019732325	4	5	2	3		5		5	7		7	26	4	5	2	3	5		5		7	7	25	29	69	22
113	161019732326	4	5	2	3				0	5		5	19	4	5	2	3			0		5	7	25	29	49	20
114	161019732327	4	5	2	3			5	5		7	7	26	4	5	2	3		5	5	7		7	25	21	59	18
115	161019732328	4	5	2	3			3	3		7	7	24	4	5	2	3		3	3	7		7	27	22	49	12
116	161019732329	4	5	2	3		7		7		6	6	27	4	5	2	3	7		7	6		4	25	29	49	31
117	161019732330	4	5	2	3		3		3		5	5	22	4	5	2	3	3		3	5		4	21	20	49	17
118	161019732331	4	5	2	3			4	4	3		3	21	4	5	2	3		4	4		3	3	23	23	69	21
119	161019732332	4	5	2	3		7		7	7		7	28	4	5	2	3	7		7		7	4	22	22	49	29
120	161019732333	4	5	2	3		5		5	3		3	22	4	5	2	3	5		5		3	3	22	24	69	23
121	161019732334	4	5	2	3			1	1	7		7	22	4	5	2	3		1	1		7	0	15	19	59	18
122	161019732335	4	5	2	3			4	4		6	6	24	4	5	2	3		4	4	6		3	21	21	69	43
123	161019732336	4	5	2	2		7		7	5		5	25	4	5	2	2	7		7		5	4	24	28	59	17
124	161019732337	4	5	2	2		3		3	3		3	19	4	5	2	2	3		3		3	7	25	22	69	41
125	161019732338	5	5	2	2		5		5	3		3	22	5	5	2	2	5		5		3	3	17	28	69	22
126	161019732339	4	5	2	3			7	7		7	7	28	4	5	2	3		7	7	7		6	23	24	79	20
127	161019732340	4	5	2	3			6	6		7	7	27	4	5	2	3		6	6	7		5	24	29	79	18
128	161019732341	4	5	2	3			7	7		6	6	27	4	5	2	3		7	7	6		6	25	29	69	12
129	161019732342	4	5	2	3		7		7	7		7	28	4	5	2	3	7		7		7	7	25	21	79	31
130	161019732343	4	5	2	3			7	7		6	6	27	4	5	2	3		7	7	6		6	27	22	69	17
131	161019732344	5	5	3	2			7	7		7	7	29	5	5	3	2		7	7	7		2	16	29	59	21
132	161019732345	5	4	3	2		4		4	7		7	25	5	4	3	2	4		4		7	7	25	20	49	29
133	161019732346	5	5	3	2		7		7		7	7	29	5	5	3	2	7		7	7		5	24	23	69	23
134	161019732347	5	5	3	2			7	7		6	6	28	5	5	3	2		7	7	6		2	21	22	59	18
135	161019732348	4	5	2	3		7		7	5		5	26	4	5	2	3	7		7		5	3	24	24	69	43
136	161019732349	4	5	2	3			5	5	7		7	26	4	5	2	3		5	5		7	7	24	19	69	17
137	161019732350	4	5	2	3			7	7	5		5	26	4	5	2	3		7	7		5	3	17	21	69	41
138	161019732351	4	5	2	3			6	6	7		7	27	4	5	2	3		6	6		7	0	17	28	49	22
139	161019732352	4	5	2	3			7	7		7	7	28	4	5	2	3		7	7	7		7	26	22	59	20
140	161019732354	4	5	2	3		6		6		1	1	21	4	5	2	3	6		6	1		7	27	28	69	18
141	161019732355	4	5	2	3		7		7	7		7	28	4	5	2	3	7		7		7	4	25	24	49	12
142	161019732356	4	5	2	2		3		3			0	17	4	5	2	2	3		3			6	26	29	59	31
143	161019732357	4	5	2	2				0		5	5	18	4	5	2	2			0	5		7	27	29	69	17
Average Marks		4.10	4.66	2.33	2.48	4.72	5.27	4.95	4.91	5.36	5.13	#REF!	4.10	4.66	2.33	2.48	4.72	5.27	4.99	4.91	5.36	5.00	22.20	22.20	56.34	32.67	

CIE (Mid Exam) CO Wise Percentage		
COURSE OUTCOME	CO Wise Sum	CO Wise Percentage %
C01	11.37	75.83
C02	12.27	81.83
C03	11.42	76.13
C04	12.13	80.89
Average	#REF!	0.00

CIE - CO Wise Sum Formula	
C01 = ASG(C01) + Q1(C01) + BestOfQ2&Q3(C01)	
C02 = ASG(C02) + Q1(C02) + BestOfQ4&Q5(C02)	
C03 = ASG(C03) + Q1(C03) + BestOfQ2&Q3(C03)	
C04 = ASG(C04) + Q1(C04) + BestOfQ4&Q5(C04)	

CIE - CO Wise Percentage	
C01 % = (C01 SUM/total C01 Marks(15))*100	
C02 % = (C02 SUM/total C02 Marks(15))*100	
C03 % = (C03 SUM/total C03 Marks(15))*100	
C04 % = (C04 SUM/total C04 Marks(15))*100	

Average Marks	32.67
Student Count >Avg	94
Total Students	143
Percentage	65.7343

SEE (End Exam) CO Wise Percentage		
C01-C04	#REF!	100.00

SEE - CO Wise Percentage	
C01-C04 = End Exam Avg Marks	

SEE - CO Wise Percentage	
C01-C04 % = (End Exam Avg Marks/70)*100	

CO ATTAINMENT					
Internal Marks %	Internal Attainment	External Marks %	External Attainment	Direct Attainment	CO Wise Attainment
C01	76	3	100.00	3	3
C02	82	3	100.00	3	3
C03	76	3	100.00	3	3
C04	81	3	100.00	3	3
Average					3

Unit I SIMPLE STRESSES AND STRAINS.

1.1 Elasticity and plasticity.

1.2 Types of stresses and strains

1.3 Hook's law

1.4 Stress - strain diagram for mild steel.
problems.

1.5 Working stress
Factor of safety

1.6 Lateral strain

1.7 Poisson's ratio

1.8 Volumetric strain.

1.9 Elastic moduli E , G and K .

Relationship between E , G and K .

1.10 Bars of tapering section.

1.11 Composite bars.

problems.

1.12. Temperature Stresses.

Problems

Strain Energy or Resilience.

Gradually applied load

Stress due to suddenly applied load.

Stress due to impact load.

problems.

Unit II

SHEAR FORCE AND BENDING MOMENT:

2.1 Definition of beam.

2.2 Types of beams.

2.3 Types of loads.

~~pro~~ Shear force

Bending moment

problems on cantilever
beams

problems on overhanging
beams.

Unit III FLEXURAL STRESSES.

3.1 Flexural Stress.

3.2 Neutral axis.

3.3 pure bending (Simple
bending)

3.4 Assumptions in simple
bending theory.

3.5 Derivation of bending
equation

Section modulus of rectang-
ular section.

Section modulus of
hollow rectangular section

→ Section modulus of hollow
circular section.
problems

Unit IV SHEAR STRESSES.

→ Derivation of formula
for shear stress.

→ Shear stress distribution
for rectangular section.

→ Shear stress distribution
for circular section.

→ Shear stress distribution
for triangular section.

→ problems.

→ Highlights

→ short questions

→ Tutorial problems.

unit V DEFLECTION OF BEAMS.

- 5.1 Beam bending into circular arc.
- 5.2 Slope, Deflection and radius of curvature.
- 5.3 Determination of slope and deflection for cantilever beams subjected to:
 - point load
 - u.d.l on whole span
 - u.v.l on whole span
- Deflection of simply supported beam subjected to
 - Central point load.
 - u.d.l
- Macaulay's Method.
- problems.
- Moment area method - Mohr's Theorem
- problems.
- Highlights
- Short questions
- Exercise questions

unit VI

PRINCIPAL STRESSES AND STRAINS.

- Introduction.
- Stresses on inclined section of a bar under axial loading.
- Normal and tangential stresses on an inclined plane for biaxial stresses.
- Two perpendicular normal stresses accompanied by a state of simple shear.
- problems.
- Graphical solution
- Mohr's circle for like stresses
- Mohr's circle for unlike stresses.
- Mohr's circle for normal stresses and shear stress.
- principal stress and principal planes.
- Mohr's circle for principal stresses.
- Theories of Failures.
- Maximum principal stress Theory

- Maximum principal Strain Theory.
- Maximum shear stress Theory.
- Maximum strain Energy theory.
- Maximum shear strain energy theory.
- Highlights
- Short questions.
- Exercise questions.

Unit VII THIN CYLINDERS.

- Thin cylinders subjected to internal pressure
- Hoop stress σ_H (circumferential stress)
- Longitudinal stress σ_L
- Circumferential strain e_c (Hoop strain)
- Longitudinal strain e_L
- Volumetric strain e_v .
- Maximum shear stress τ_{max} .
- Thin spherical shells.
- Problems.
- Short Questions.
- Exercise Questions.

UNIT VIII

THICK CYLINDERS.

- Introduction.
- Lame's Theory.
 - (i) Radial pressure
 - (ii) Hoop pressure.
- problems
- Compound cylinders.
- Difference of radii for shrinkage.
- problems.
- Thick spherical Shells.
- problems.
- Short Questions.
- Exercise Questions.

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Strength Of Materials - I

Unit - I Simple Stresses and Strains :

1.1.2 Elasticity and plasticity :

Elasticity is the property of a material by which the material regains its original shape and size after removal of external forces.

For example a rubber band elongates on applying tensile force and goes back to its original shape and size on removal of force, therefore it is an elastic material. The other common elastic material is steel.



Fig 1.1.2

ORIGINAL LENGTH



ELONGATED STATE

OR ORIGINAL LENGTH

1.1.2 Plasticity is the property of a material by which it does not regain its original shape and size after removal of external forces. For example clay, used for making toys, is a plastic material. It does not regain its original shape and size after removal of external forces.

CLAY

IN ROLLING

1.2.a Types of Stresses and Strains

STRESS

Stress may be defined as $\frac{\text{Load}}{\text{Area}} = \frac{P}{A}$, The various

types of stresses are

(i) Tensile stress $\sigma_t = \frac{\text{Tensile force}}{\text{Area of cross section}} = \frac{P_t}{A}$

(ii) Compressive stress $\sigma_c = \frac{\text{Compressive force}}{\text{Area of cross section}} = \frac{P_c}{A}$

(iii) Shear stress $\tau = \frac{\text{Shear force}}{\text{Area of cross section}} = \frac{P_s}{A}$

(iv) Torsional Stress τ (It is determined using torsional equation)

(v) Bending Stresses or Flexural Stresses f_b (It is determined using bending equation which is studied in unit III)

STRAIN

1.2.b Strain is the ratio of change in length to original length.

Strain is generally denoted as 'e'

$$\text{Strain } e = \frac{\text{Change in length}}{\text{original length}}$$

If a member of length 'l' is subjected to external load 'P' which causes change in

length δl then

$$e = \frac{\delta l}{l}$$

↳ 1.3.a Hook's law It states that within elastic limit

• stress is proportional to strain

Let σ = stress in any member

e = strain in the member.

As per Hook's law stress $\propto$ strain

$$\sigma \propto e$$

$$\text{or } \sigma = \text{constant} \times e$$

where constant = E (modulus of elasticity)

$$\sigma = E \times e$$

$$\text{or } \boxed{E = \frac{\sigma}{e}} \quad \text{Eqn (1.3.a) ie Modulus of}$$

elasticity is the ratio of stress to strain.

$$\boxed{E = \frac{\text{STRESS}}{\text{STRAIN}}}$$

1.3.b change in length δl of any member may be determ.

ined using Eqn (1.3.a)

$$\text{Modulus of elasticity } E = \frac{\sigma (\text{stress})}{e (\text{strain})}$$

$$\text{where stress } \sigma = \frac{P (\text{Load})}{A (\text{Area})}$$

$$\text{and strain } e = \frac{\delta l (\text{change in length})}{l (\text{original length})}$$

Substitute σ and e in Eqn 1.3.a

$$E = \frac{P/A}{\delta l/l} = \frac{P}{A} \times \frac{l}{\delta l}$$

Rearranging $\delta l = \frac{Pl}{AE}$ ————— Eqn (1.3.6)

1.4. Stress-Strain Diagram for mild steel

The pictorial graph obtained by taking stress on y-axis and strain on x-axis is known as Stress-strain diagram. steel being used commonly by engineers its stress-strain diagram is studied in detail.

Tensile test is generally carried out on circular bar of uniform cross section. The applied load is gradually increased from zero (in small increments) till the specimen fails. At each step stress σ and corresponding change in length (δl) is measured. Strain is calculated from the change in length. stress-strain is plotted in the form of graph and modulus of elasticity (E) may be determined from the graph.

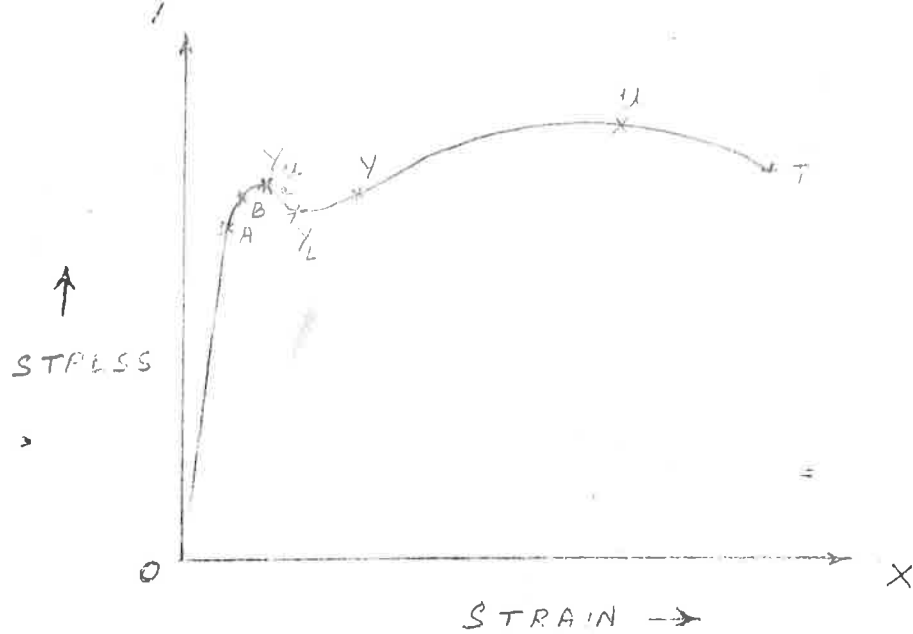


Fig 1.4.a STRESS - STRAIN DIAGRAM OF MILD STEEL.

Initially stress-strain diagram is linear from 'O' to 'A', upto this point i.e. 'A' stress is proportional to strain i.e. Hook's law is obeyed. The stress upto which Hook's law is obeyed is known as "limit of proportionality" which is from O to A.

The material remain elastic (it regains its original shape if the force is removed) upto point 'B'. The stress upto which the material remain elastic is known as elastic limit.

If the stress is increased beyond point 'B' sudden increase in length is observed i.e. the material "yields". This stress is known as "upper yield point" and denoted by Y_u in diagram.

Residual Strain : In between point B and γ_u the material undergoes a permanent strain known as residual strain.

The specimen experience a drop in stress which is denoted by γ_L i.e. lower yield point. The stress-strain curve is unsteady between γ_L and γ .

Strain hardening : Increase in the load beyond point 'Y' causes further strain in the material which is known as strain hardening.

The stress at 'u' is known as ultimate stress, which is the maximum stress.

On further increase in load the specimen elongates with slight decrease in stress. The specimen contracts i.e. diameter is reduced (it is known as neck formation), the specimen fractures (fails) at point 'f'. The fractured pieces form cup and cone shape.



NECK FORMATION

fig 1.4.b



CUP AND CONE FAILURE

fig 1.4.c

- 1@ what do you understand by yielding, strain hardening, neck formation and permanent set.
- ⑥ Find the minimum dia of a steel wire with which a load of 4000 N can be raised so that the tensile stress in the wire may not exceed 130 N/mm^2 . calculate the extension of the wire, if it is 3 m long. Take $E = 200 \text{ GPa}$.

Ans:

(i) yielding: If a material is stressed beyond elastic limit a sudden increase in length is observed in the material. This behaviour of the material is known as yielding.

(ii) Strain Hardening: If a material is stressed beyond yield point a large amount of strain (change in length) is observed. This strain is known as strain hardening.

(iii) Neck formation: when the specimen is stressed beyond ultimate stress it fractures (fails). Before failure reduction in diameter of the specimen is observed. This reduction in dia is known as neck formation.

(iv) Permanent Set: In between elastic limit (point B in stress-strain curve) and upper yield point γ_u the material undergoes a permanent change in length (permanent strain) which is known as permanent ~~set~~ set.

(b) Solution dia $d = ?$; Load $P = 4000 \text{ N}$,

tensile stress $\sigma_f = 130 \text{ N/mm}^2$ $\delta l = ?$

Length $l = 3 \text{ m} = 3000 \text{ mm}$, $E = 200 \text{ GPa}$
 $= 200 \times 1000 \text{ MPa}$
 $= 2 \times 10^5 \text{ N/mm}^2$

→ We know stress $\sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$

$$130 = \frac{4000}{\frac{\pi d^2}{4}}$$

$$130 = \frac{16000}{\pi d^2}$$

$$\boxed{d = 6.26 \text{ mm}}$$

$$[\text{Area } A = \frac{\pi d^2}{4}]$$

$d = \text{dia of bar}$

→ To find change in length δl :

we know Modulus of Elasticity $E = \frac{\text{Stress}}{\text{Strain}}$

$$\text{or } E = \frac{\text{Load}}{\delta l \times \text{Area}}$$

$$\text{change in length } \delta l = \frac{Pl}{AE} = \frac{4000 \times 3000}{\frac{\pi \times 6.26^2 \times 2 \times 10^5}{4}}$$

$$= \frac{48 \times 10^6}{\pi \times 6.26^2 \times 2 \times 10^5}$$

$$\boxed{\delta l = 0.16 \text{ mm}}$$

1.95 mm

Ultimate Strength:

$$\begin{aligned}\text{ultimate strength} &= \frac{\text{Load at fracture}}{\text{Area}} \\ &= \frac{40 \times 10^3}{113.09} \\ &= \underline{\underline{353.70 \text{ N/mm}^2}}\end{aligned}$$

Breaking Strength:

$$\begin{aligned}\text{Breaking strength} &= \frac{\text{Max. load}}{\text{Area}} \\ &= \frac{60 \times 10^3}{113.09} \\ &= \underline{\underline{530.55 \text{ N/mm}^2}}\end{aligned}$$

$$\text{percentage elongation} = \frac{\delta l}{l} \times 100.$$

Total elongation δl = $\frac{Pl}{AE}$
at 60 kN load.

$$= \frac{60 \times 10^3 \times 50}{113.09 \times 2.52 \times 10^5}$$

$$= \underline{\underline{0.105 \text{ mm.}}} \quad \text{--- Substitute in above equation}$$

$$\text{percentage elongation} = \frac{0.105}{50} \times 100.$$

$$= \underline{\underline{0.21\%}}$$

$$\text{percentage elongation} = \frac{\delta l_{\text{final}}}{l} \times 100$$

$$= \frac{(70 - 50)}{50} \times 100$$

$$= \underline{\underline{40\%}}$$

A specimen of material having having 12mm dia. is tested under tension over a gauge length of 50mm. At 20 kN load the extension was 0.035 mm. The max. load taken by the specimen was 60 kN and the fracture occurred at 40 kN. Find the modulus of elasticity, ultimate strength, breaking strength and percentage elongation, if the final length of the specimen was 70 mm.

Solution:

$$\text{dia } d = 12 \text{ mm} \quad ; \quad l = 50 \text{ mm} \Rightarrow A = \frac{\pi \times 12^2}{4}$$

$$= \underline{113.09 \text{ mm}^2}$$

$$\text{when } P = 20 \text{ kN} \quad \delta l = 0.035 \text{ mm.}$$

$$\text{Max load taken } P_{\max} = 60 \text{ kN}$$

$$\text{Fracture load } P_{fn} = 40 \text{ kN.}$$

$$\text{Final length} = 70 \text{ mm.}$$

To find mod. of elasticity E

$$\text{Mod. of elasticity } E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\delta l/l} = \frac{P}{A} \div \frac{\delta l}{l}$$

$$= \frac{P}{A} \times \frac{l}{\delta l}$$

$$= \frac{20 \times 10^3}{113.09} \times \frac{50}{0.035}$$

$$= \underline{2.52 \times 10^5 \text{ N/mm}^2}$$

Set No. 1

State Hook's law. Sketch the stress-strain diagram for a ductile material like mild steel tested under tension upto destruction, marking the salient points on it. Explain the significance of each point.

Solution:

1) @ Hook's law: It states that within elastic limit stress is proportional to strain.

Stress $\propto$ Strain (within elastic limit)

Stress = Constant $\times$ Strain

$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$.

This constant is known as modulus of elasticity E

$$E = \frac{\text{Stress}}{\text{Strain}}$$

(b) Stress-strain ~~retd~~ diagram of mil steel

Refer Section 1.4 page (4) and (5).

1

2

1.5.a WORKING STRESS σ_w

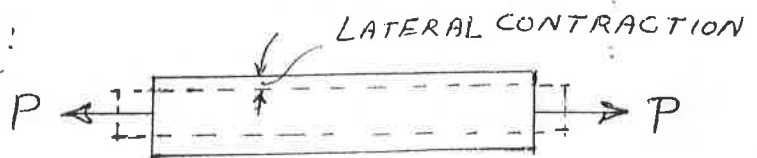
Working stress is the maximum permissible of a material obtained by dividing its yield stress by suitable factor of safety.

$$\sigma_w = \frac{\text{Yield Stress}}{\text{Factor of Safety}} = \frac{\sigma_y}{F.S.}$$

1.5.b Factor of Safety F.S $F.S = \frac{\text{Yield Stress}}{\text{Working Stress}}$

In some materials like concrete and non-ferrous alloys stresses are not directly proportional to strains. To account for this drawback, the yield stress σ_y of the material is divided by a factor to obtain its allowable stress, this factor is known as factor of safety.

1.6.a LATERAL STRAIN:



A member subjected to tensile force elongates, simultaneously its dimensions are reduced in lateral direction. This change ^{in dimensions} along lateral direction is known as lateral strain.

Ex: A rubber band when pulled elongated, at the same time its lateral dimensions (thickness) is reduced, which is lateral strain.

1.7 Poisson's Ratio: $\frac{1}{m}$:

It is defined as lateral strain divided by longitudinal strain

$$\frac{1}{m} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

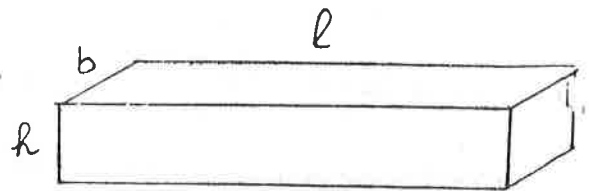
Note: Max. value of Poisson's ratio $\frac{1}{m} = 0.5$ for rubber

Min. value of Poisson's ratio $\frac{1}{m} = 0$ for cork.

Poisson's ratio of steel $\frac{1}{m} = 0.33$.

1.8 Volumetric Strain: e_v :

$$e_v = \frac{\text{change in volume}}{\text{original volume}}$$



→ volumetric strain of a rectangular bar $e = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta h}{h}$

→ volumetric strain of a cylindrical rod $e = \frac{\delta l}{l} + 2 \frac{\delta d}{d}$

→ volumetric strain of a sphere $e = 3 \frac{\delta d}{d}$ ($d = \text{dia}$)

1.9 Q: How many elastic moduli are there? what are they?

Ans: There are three elastic moduli, they are

① Modulus of elasticity $E = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

② Modulus of Rigidity $G = \frac{\text{Shear stress}}{\text{Transverse strain}}$

③ Bulk Modulus $K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$

Elastic Modulus = The ratio of force exerted upon a substance or body to the resultant deformation

1.9(b) Relationship between E, G and K :

$$\text{Mod. of elasticity } E = 2G \left(1 + \frac{1}{m}\right) \quad \text{--- (1)}$$

$$\text{Mod. of elasticity } E = 3K \left(1 - \frac{2}{m}\right)$$

$$\text{Mod. of elasticity } E = \frac{9KG}{3K + G}$$

where G = Mod. of rigidity

K = Bulk Modulus

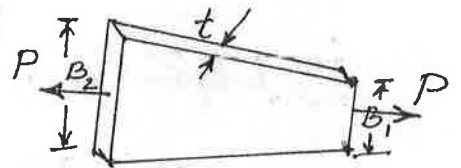
$\frac{1}{m}$ = Poisson's ratio

Tapering of Rod = Reduce in thickness towards one end.

1.10 Bars of tapering Section

Elongation of a bar of uniform thickness and tapering section is obtained from the equation

$$\text{Elongation } \delta l = \frac{Pl}{(B_2 - B_1)tE} \log\left(\frac{B_2}{B_1}\right)$$



where P = Force applied.

B_2 = Larger width of bar.

B_1 = Smaller width of bar.

t = Thickness of bar.

1.11 Composite Bars :

When a bar is made up of two different materials it is known as composite bar.



Consider the composite bar shown in figure above made of steel and copper subjected to a compressive force 'P'.

Two equations may be formulated

Firstly : Total load = Load taken by steel (P_s) + Load taken by copper (P_c)

$$P = P_s + P_c$$

$$\boxed{P = \sigma_s A_s + \sigma_c A_c} \quad \text{--- (1)}$$

where P = Applied load.

σ_s & σ_c = Stress in steel and stress in copper.

A_s & A_c = Area of steel bar and area of copper bar respectively.

* Secondly :

Change in length of steel δl = Change in length of copper.

i.e. $\delta l_{\text{steel}} = \delta l_{\text{copper}}$

$$\frac{P_s l_s}{E A_s} = \frac{P_c l_c}{E A_c} \rightarrow \boxed{\sigma_s l_s = \sigma_c l_c} \quad \text{--- (2)}$$

using Eqn (1) & (2) stresses σ_s & σ_c are obtained.

Prob 4: JNTU, B.Tech - I Sem. Supp. May/June 2009.
100 mm long and

A bar of cross-section 8 mm x 8 mm is subjected to an axial pull of 7000 N. The lateral dimension of the bar is found to be changed to 7.9985 mm x 7.9985 mm. If the modulus of rigidity of the material is $0.8 \times 10^5 \text{ N/mm}^2$, determine the poisson's ratio and modulus of elasticity if the bar elongates by 0.05 mm. (16 marks)

Solution:

Size of the bar 8 mm x 8 mm.

Reduced Size of the bar 7.9985 mm x 7.9985 mm.

$$\begin{aligned}\text{Change in lateral dimensions} &= (8 - 7.9985) \\ &= \underline{1.5 \times 10^{-3} \text{ mm}}\end{aligned}$$

Given Axial pull applied $P = 7000 \text{ N}$.

Given Modulus of rigidity $G = 0.8 \times 10^5 \text{ N/mm}^2$

$$\begin{aligned}\text{Lateral strain} &= \frac{\text{Change in lateral dimension}}{\text{Original dimension}} \\ &= \frac{1.5 \times 10^{-3}}{8} \\ &= 1.875 \times 10^{-4}\end{aligned}$$

$$\text{Longitudinal Strain} = \frac{\Delta l}{l} = \frac{0.05}{100} = 0.0005$$

$$\text{Poisson's ratio } \frac{1}{m} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = \frac{1.875 \times 10^{-4}}{0.0005}$$

$$\boxed{\frac{1}{m} = 0.375}$$

To find mod. of elasticity E

we know $E = 2G \left(1 + \frac{1}{m}\right)$

$$= 2 \times 0.3 \times 10^5 \left(1 + 0.375\right)$$

$$= \underline{\underline{2.2 \times 10^5 \text{ N/mm}^2}}$$

Prob 5

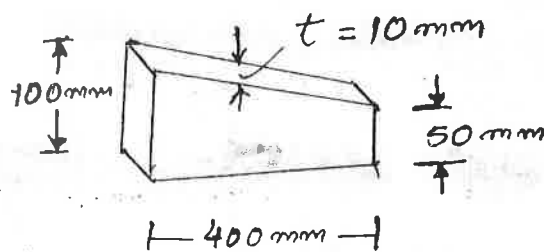
The extension ~~of~~ ⁱⁿ a rectangular ^{steel} bar of length 400 mm and thickness 10 mm is found to be 0.21 mm.

The bar tapers uniformly in width from 100 mm to 50 mm. If E for the bar is $2 \times 10^5 \text{ N/mm}^2$, determine the axial load on the bar. (16 marks).

Solution:

$$\delta l = 0.21 \text{ mm}$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$



Elongation of a tapering rod $\delta l = \frac{Pl}{(B_2 - B_1)t \cdot E} \times \log_e \frac{B_2}{B_1}$

where B_2 = Larger width

$$= 100 \text{ mm}$$

B_1 = Smaller width

$$= 50 \text{ mm}$$

Thickness $t = 10 \text{ mm}$

given $\delta l = 0.21 \text{ mm}$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

Substitute the above values in standard equation

$$0.21 = \frac{P \times 400}{(100 - 50) 10 \times 2 \times 10^5} \times \log_e \left(\frac{100}{50} \right)$$

note: $\log_e \frac{100}{50}$ is natural log which is shown as "ln" in calculator.

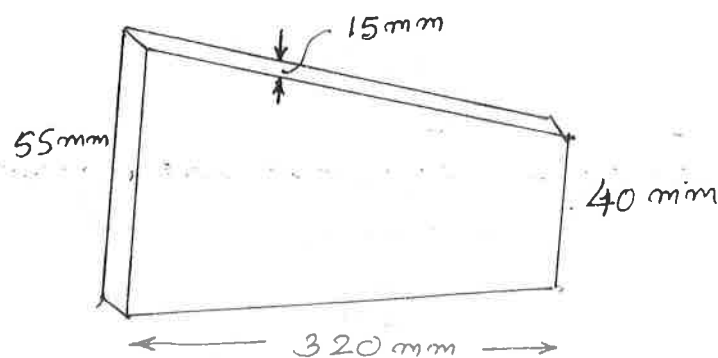
$$0.21 = \frac{400P}{500 \times 2 \times 10^5} \times 0.693$$

$$P = \underline{\underline{75757 \text{ N}}}$$

Prob 6: A flat steel bar is of thickness 15 mm and tapers uniform from ~~an~~ width of 55 mm to 40 mm. It is subjected to a tensile force of 110 kN, determine the change in length of the bar if the original length is 320 mm, Take $E = 2 \times 10^5 \text{ N/mm}^2$.

Solution:

Change



(20)

$$\left. \begin{array}{l} \text{Change in length} \\ \text{of bar} \end{array} \right\} \delta l = \frac{Pl}{(B_2 - B_1)t.E} \log\left(\frac{B_2}{B_1}\right)$$

Given $t = 15 \text{ mm}$; $B_2 = 55 \text{ mm}$; $B_1 = 40 \text{ mm}$

$P = 110 \text{ kN}$; $l = 320 \text{ mm}$; $E = 2 \times 10^5 \text{ N/mm}^2$

$$\boxed{\delta l = ?}$$

Substitute the given values in standard equation

$$\delta l = \left[\frac{110 \times 10^3 \times 320}{(55 - 40) 15 \times 2 \times 10^5} \right] \log\left(\frac{55}{40}\right)$$

$$= [0.78] \times 0.32$$

$$\boxed{\delta l = 0.25 \text{ mm}}$$

Prob 7: problem on poisson's ratio and elastic moduli E , G & K .

A metallic circular bar of length 300 mm and dia 30 mm is subjected to an axial load of 50 kN . The bar elongates by 0.10 mm and the dia reduces by 0.0036 mm . Calculate

- (a) Modulus of Elasticity ' E ' (b) Mod. of rigidity ' G '
(c) Bulk Modulus ' K ' (d) poisson's ratio. $\frac{1}{m}$

Solution: $l = 300 \text{ mm}$; $d = 30 \text{ mm}$; $P = 50 \text{ kN}$

$\delta l = 0.10 \text{ mm}$; $\delta d = 0.0036 \text{ mm}$

Area of bar $A = \frac{\pi d^2}{4} = \frac{\pi \times 30^2}{4}$

$= 706.86 \text{ mm}^2$

Axial Stress or longitudinal stress $\sigma = \frac{P}{A}$

$$\text{or } \sigma = \frac{50 \times 10^3}{706.86}$$
$$= \underline{\underline{70.73 \text{ N/mm}^2}}$$

Linear
Axial or longitudinal strain $e = \frac{\delta l}{l}$

$$\text{or } e = \frac{0.10}{300} = \underline{\underline{3.33 \times 10^{-4}}}$$

Mod of elasticity $E = \frac{\text{Axial Stress}}{\text{Axial Strain}}$ or $\frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$

$$= \frac{70.73}{3.33 \times 10^{-4}}$$

$$\boxed{E = 2.12 \times 10^5 \text{ N/mm}^2}$$

① Poisson's ratio $\frac{1}{m} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

$$\rightarrow \text{Lateral strain } e_{\text{lat}} = \frac{\delta d}{d} = \frac{0.0036}{30}$$
$$= \underline{\underline{1.2 \times 10^{-4}}}$$

Substitute in above Equation

$$\rightarrow \text{Poisson's ratio } \frac{1}{m} = \frac{1.2 \times 10^{-4}}{3.33 \times 10^{-4}}$$

$$\boxed{\frac{1}{m} = 0.36}$$

(22)
b) Mod of rigidity G:

we know $E = 2G \left(1 + \frac{1}{m}\right)$

Substitute $E = 2.12 \times 10^5 \text{ N/mm}^2$ and $\frac{1}{m} = 0.36$

$$\Rightarrow 2.12 \times 10^5 = 2 \times G (1 + 0.36)$$

$$G = 0.78 \times 10^5 \text{ N/mm}^2 \quad \checkmark$$

c) Bulk Modulus K:

we know $E = 3K \left(1 - \frac{2}{m}\right)$

Substitute E and $\frac{1}{m}$ in above equation

$$2.12 \times 10^5 = 3K (1 - 2 \times 0.36)$$

$$= 3K \times 0.28$$

$$K = 2.52 \times 10^5 \text{ N/mm}^2 \quad \checkmark$$

1.12 Temperature Stresses:

→ When the temperature of a material is increased or decreased its dimensions also increase or decrease respectively. If the material is allowed to expand or contract no temperature stresses are developed.

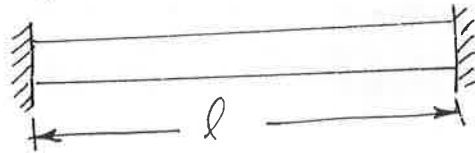
→ If the material is not allowed to expand or contract (i.e. restrained) then temperature stresses are developed.

→ If temperature is raised then compressive stresses are developed.

If the temperature is reduced then tensile stresses are developed.

1.13 Magnitude of Temperature Stress:

Consider a bar of length l fixed between two fixed supports



Let l = Length of the bar

t_1 = Initial temperature

t_2 = Final temperature

$t = (t_2 - t_1)$ change in temperature

α = Coefficient of expansion of the material of bar

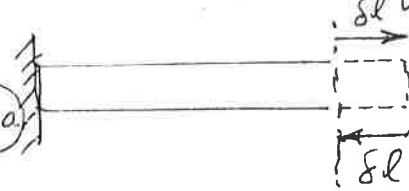
(24)
 A = Area of cross section of bar

E = Modulus of elasticity

→ If the bar is free to expand, the change in length of the bar

$$\delta l = \text{Coeff of expansion per degree per mm} \times \text{Length of member} \times \text{Temp. change}$$
$$= \alpha l t$$

$\delta l = l \alpha t$

1.13 a 

→ The member is not permitted to expand by developing a compressive stress $\sigma_t = \frac{P}{A}$ where

P = Force developed.

Now change in length due to developed force

$\delta l = \frac{P \cdot l}{A E}$

 , this change is equal to the change in temperature. 1.13 b

Equating the above two equations of δl .

$$\left(\frac{P}{A E} \right) l = l \alpha t$$

$$\frac{\sigma_t}{E} = \alpha t$$

Temperature Stress

$\sigma_t = E \alpha t$

1.13 c

Rails of 15m length were laid on the track when the temperature was 20°C . A gap of 1.8mm was kept between two consecutive rails. At what max. temperature the rails will remain stress free? If the temperature is raised further by 15°C what will be the magnitude and nature of stress induced in the rails? Take $\alpha_s = 12 \times 10^{-6}/^{\circ}\text{C}$ and $E_s = 200 \text{ GPa}$

Solution

$$l = 15\text{m} ; t_1 = 20^{\circ}\text{C} , \quad \delta l = 1.8\text{mm} \quad t_2 = ?$$

$$\text{Let change in temp} = t ; \quad E_s = 200 \times 10^3 \text{ N/mm}^2$$

→ The rails will remain stress free till the gap of 1.8mm is filled.

$$\delta l = l \alpha t \quad \text{Substitute } \delta l, l \text{ and } \alpha$$

$$1.8 = 15 \times 10^3 \times 12 \times 10^{-6} \times t$$

$$\boxed{t = 10^{\circ}\text{C.}}$$

The temperature may be raised to $(20^{\circ} + 10^{\circ}) = \underline{\underline{30^{\circ}\text{C}}}$

(b) When the temperature is raised by 15°C .

$$\text{Stress } \sigma_t = E \alpha t \quad \text{Substituting } E, \alpha \text{ and } t.$$

$$= 200 \times 10^3 \times 12 \times 10^{-6} \times 15$$

$$\boxed{\sigma_t = 36 \text{ N/mm}^2}$$

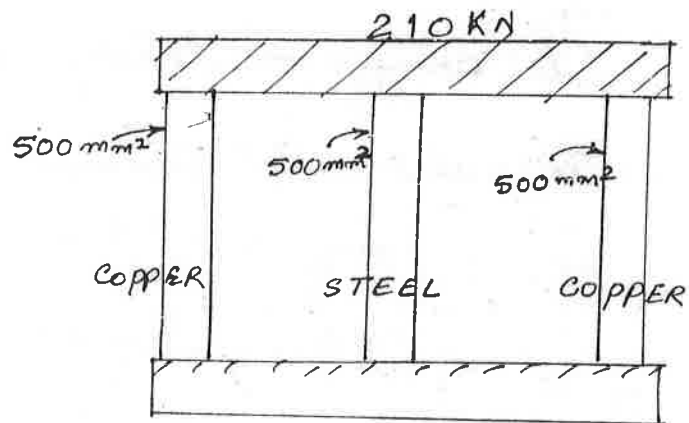
Prob: 9 (26) Set No: 1 Code No: R050210101)

A weight of 210 kN is supported by three short pillars each of sectional area 500 mm². The central pillar is of steel and the outer two are of copper. The pillars are so adjusted that at a temperature of 15°C each carries equal load. The temperature is then raised to 95°C. Find the stress in each pillar at 15°C and 95°C. Take $E_s = 200 \text{ GPa}$ and $E_c = 80 \text{ GPa}$. $\alpha_s = 12 \times 10^{-6} / ^\circ\text{C}$ and $\alpha_c = 18 \times 10^{-6} / ^\circ\text{C}$. (16 marks).

Solution

Area $E_s = 200 \times 10^3 \text{ N/mm}^2$

$E_c = 80 \times 10^3 \text{ N/mm}^2$



Stresses at 15°C.

→ Given load is shared equally by three pillars } $P = \frac{210}{3} = \underline{\underline{70 \text{ kN}}}$

→ Stress in steel } = stress in copper } = $\frac{P}{A}$
due to load.

$\sigma_s = \sigma_{cu} = \frac{70 \times 10^3}{500}$

$= \underline{\underline{140 \text{ N/mm}^2}}$

Stress at 95°C

Change in temperature $t = t_2 - t_1 = 95 - 15$
 $= \underline{\underline{80^\circ\text{C}}}$

Temperature
1 Stress in steel pillar $\sigma_{t-s} = E_s \alpha_s t$
 $= 200 \times 10^3 \times 12 \times 10^{-6} \times 80$
 $= \underline{\underline{192 \text{ N/mm}^2}}$

Temperature
1 Stress in copper pillar $\sigma_{t-c} = E_c \alpha_c t$
 $= 80 \times 10^3 \times 18 \times 10^{-6} \times 80$
 $= \underline{\underline{115.2 \text{ N/mm}^2}}$

Prob 10:

In a railway line rails are 40 m long are laid at 300°C . Calculate the stress if the temperature is raised to 350°C and there is no allowance for expansion. What should be the permissible allowance to have zero stress. Take $E = 200 \text{ GPa}$ and $\alpha = 12 \times 10^{-6}/^\circ\text{C}$

Solution: $l = 40 \text{ m}$; $t_1 = 300^\circ\text{C}$; $t_2 = 350^\circ\text{C}$

$E = 200 \times 10^3 \text{ N/mm}^2$; $\alpha = 12 \times 10^{-6}/^\circ\text{C}$

$t = (t_2 - t_1) = (350 - 300) = \underline{\underline{50^\circ\text{C}}}$

Temperature
a) Stress in rails $\sigma_t = E \alpha t$
 $= 200 \times 10^3 \times 12 \times 10^{-6} \times 50$
 $= \underline{\underline{120 \text{ N/mm}^2}}$

b) Permissible allowance $= \delta l = l \alpha t$
 $= 40 \times 10^3 \times 12 \times 10^{-6} \times 50$
 $= \underline{\underline{24 \text{ mm}}}$

L

L

Unit I SHORT QUESTIONS

- ① Stress is defined as
(a) Load $\times$ area (b) $\frac{\text{Load}}{\text{area}}$ (c) $\frac{\text{Area}}{\text{Load}}$ (d) None.
- ② Strain is defined by the ratio
(a) change in length $\times$ original length (b) $\frac{\text{change in length}}{\text{original length}}$
(c) original length / change in length (d) None.
- ③ If a wire is subjected to a load 'P' the change in length δl is given by _____. Given length of wire 'L', area of cross section 'A' and modulus of elasticity E.
(a) $\delta l = \frac{AL}{P.E}$ (b) $\delta l = \frac{PL}{AE}$ (c) $\delta l = \frac{AE}{PL}$ (d) None.
- ④ Poisson's ratio $\frac{1}{m}$ is defined as:
(a) $\frac{\text{Longitudinal Strain}}{\text{Lateral Strain}}$ (b) $\frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$
(c) Lateral Strain $\times$ Longitudinal Strain (d) Either (a) or (b).
- ⑤ Modulus of elasticity or Young's Modulus of elasticity is defined as
(a) Stress $\times$ strain (b) $\frac{\text{Stress}}{\text{Strain}}$ (c) $\frac{\text{Strain}}{\text{Stress}}$ (d) None.
- ⑥ Modulus of rigidity C, G or N is defined as
(a) $\frac{\text{Shear Stress}}{\text{Axial Stress}}$ (b) $\frac{\text{Axial Stress}}{\text{Shear Stress}}$ (c) $\frac{\text{Shear Stress}}{\text{Corresponding strain}}$

⑦ Modulus of elasticity 'E', modulus of rigidity G and poisson's ratio $\frac{1}{m}$ are related as

(a) $E = 2G(1 + \frac{1}{m})$ (b) $E = 3K(1 - \frac{2}{m})$ (c) $E = \frac{9KG}{3K + G}$

(d) All the above.

⑧ Maximum value of poisson's ratio is nearly equal to

(a) 1.0 (b) 0.33 (c) 0.5 (d) Either (b) or (c).

⑨ If a metallic bar of length 'l' coefficient of expansion ' α ' is subjected to change in temperature 't', then change in length δl is given by the expression

(a) $E \propto t$ (b) $l \propto t$ (c) $\frac{l \alpha}{t}$ (d) $\frac{\alpha t}{l}$

⑩ If a member is ~~restrained~~ not permitted to expand and subjected to a change in temperature 't' on a length 'l' then stress due to change in temperature is given by the expression

(a) $E \propto t$ (b) $l \propto t$ (c) $\frac{l \alpha}{t}$ (d) $\frac{E \alpha t}{l}$

where E = Modulus of elasticity of steel.

STRAIN ENERGY OR RESILIENCE:

~~It is defined as~~ The work done on a member is stored by the member as energy, ~~this~~ ^{this} stored energy is called STRAIN ENERGY - OR RESILIENCE.

Strain Energy or Resilience = work done on the member.

To find Strain Energy stored per unit volume:

Consider the member shown in figure

U = strain Energy stored in the member.

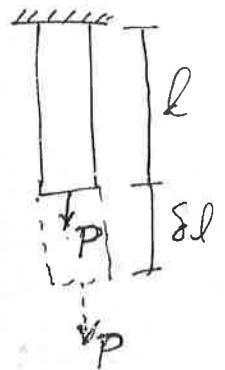
Let l = length of the member

A = Area of cross-section

δl = change in length

P = Force applied.

$\sigma = \frac{P}{A}$ Stress developed



Strain energy stored in the member = Average resistance $\times$ displacement

$$U = \frac{P}{2} \times \delta l$$

$$= \frac{P}{2} \times \frac{Pl}{AE}$$

Substituting $P = \sigma \times A$.

$$= \frac{\sigma \times A}{2} \times \frac{\sigma \times l}{E}$$

$$= \frac{\sigma^2}{2E} \times A \cdot l \quad (\text{But } A \times l = V \text{ i.e. volume})$$

$$\boxed{U = \frac{\sigma^2}{2E} \times V} \quad \text{N-mm}$$

$\therefore$ Strain energy per unit volume = $\frac{\sigma^2}{2E} \times \frac{V}{V}$

$$\boxed{u' = \frac{\sigma^2}{2E}}$$

STRESS DUE TO

- ① Gradually Applied Load
- ② Suddenly Applied Load
- ③ Impact Loading.

① Gradually Applied load

- Equating strain energy stored and work done
- Consider a ~~member~~^{rod} of length 'l' c.s area 'A' subjected to stress σ_g and change in length δl , and applied force is 'P'.

$$\left. \begin{array}{l} \text{Strain energy stored} \\ \text{in the member} \end{array} \right\} = \frac{\sigma^2}{2E} \times \text{Volume of rod}$$

$$= \frac{\sigma^2}{2E} \cdot Al. \quad \text{--- (1)}$$

$$\left. \begin{array}{l} \text{work done by the} \\ \text{applied load} \end{array} \right\} = \text{Average load} \times \text{extension}$$

$$= \frac{P}{2} \times \delta l. \quad \text{--- (2)}$$

Equating Eqn (1) & Eqn (2).

$$\frac{\sigma^2}{2E} \cdot Al = \frac{P}{2} \delta l. \quad \left(\text{But } \delta l = \frac{Pl}{AE} \right)$$

$$\frac{\sigma^2}{2E} \cdot Al = \frac{P}{2} \cdot \frac{\sigma l}{E} \Rightarrow \sigma A = P$$

⇒ Stress $\boxed{\sigma_g = \frac{P}{A}}$

stress due to gradually applied load $\boxed{\sigma_g = \frac{P}{A}}$

② Stress due to Suddenly Applied load

Let the same load 'P' be suddenly applied on the rod.

$$\rightarrow \text{strain energy stored} = \frac{\sigma^2}{2E} \times Al. \quad \text{--- (1)}$$

$$\begin{aligned} \rightarrow \text{work done} &= \text{Applied load} \times \text{displacement} \\ &= P \times \delta l \\ &= P \times \frac{\sigma \cdot l}{E} \quad \text{--- (2)} \end{aligned}$$

Equating (1) and (2).

$$\frac{\sigma^2}{2E} \times Al = P \times \frac{\sigma \cdot l}{E}$$

$$\boxed{\sigma_s = 2 \cdot \frac{P}{A}}$$

where σ_s = Stress due to suddenly applied load.

③ STRESS DUE TO IMPACT LOAD:

Let the load 'P' is dropped from a height 'h'.

$$\rightarrow \text{strain energy} = \text{work done}$$

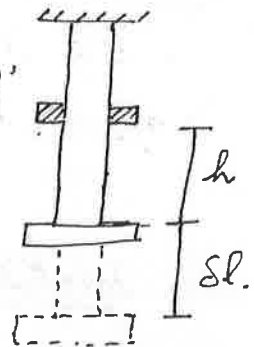
$$\frac{\sigma^2}{2E} \times Al = P(h + \delta l) \left[\text{potential Energy} \right]$$

$$= P \left(h + \frac{\sigma l}{E} \right)$$

$$\frac{\sigma^2}{2E} \cdot Al - \frac{P \cdot \sigma \cdot l}{E} = P \cdot h \quad \Rightarrow \quad \sigma_i = \sqrt{\frac{2PhE}{Al}}$$

Adding $\frac{\sigma^2}{A^2}$ on both sides and solving

$$\delta l \ll h \text{ or } \delta l \ll h$$



prob: 11

(JNTU - May/June 2009 - Suppl. Set NO: 1
Code NO: 43001)

The maximum instantaneous extension, produced by an unknown falling weight through a height of 4cm in a vertical bar of length 3m and of cross-sectional area 5cm^2 , is 2.1mm.

Determine (a) the instantaneous stress induced in the vertical bar.

(b) value of unknown weight

Take $E = 2 \times 10^5 \text{ N/mm}^2$. (16 marks)

Solution: $h = 4\text{cm}$; $l = 3\text{m}$; $A = 5\text{cm}^2$
 $= 40\text{mm}$; $= 3 \times 10^3\text{mm}$; $= 5 \times 10^2\text{mm}^2$

$$\delta l = 2.1\text{mm}$$

(a) Instantaneous stress, σ_i

change in length $\delta l = \left(\frac{P_i l}{A E} \right) = \frac{\sigma_i l}{E}$

$$2.1 = \frac{\sigma_i \times 3 \times 10^3}{2 \times 10^5}$$

Instantaneous Stress $\sigma_i = 140 \text{ N/mm}^2$ ✓

(b) Instantaneous stress $\sigma_i = \sqrt{\frac{2 P h E}{A l}}$

$$140 = \sqrt{\frac{2 P \times 40 \times 2 \times 10^5}{5 \times 10^2 \times 3 \times 10^3}}$$

$$P = 1836.9 \text{ N}$$

If a tensile load of 50 kN is applied suddenly to a circular bar of 4 cm diameter and 5 m long.

Determine (a) Maximum instantaneous stress induced.

(b) Instantaneous ~~stress~~ elongation in the rod.

(c) Strain energy absorbed in the rod.

Take $E = 2 \times 10^5 \text{ N/mm}^2$. (16 marks)

SOLUTION: $P = 50 \text{ kN}$; dia = 4 cm = 40 mm
 $l = 5 \text{ m} = 5 \times 10^3 \text{ mm}$.

$$\text{Area } A = \frac{\pi d^2}{4} = \frac{\pi \times 40^2}{4} = \underline{1256.63 \text{ mm}^2}$$

(a) Instantaneous stress due to suddenly applied load $\left\{ \sigma_s = \frac{2P}{A} = \frac{2 \times 50 \times 10^3}{1256.63} \right.$

$$\sigma_s = \underline{79.58 \text{ N/mm}^2}$$

(b) Instantaneous elongation $\delta l = \left(\frac{P_i}{A} \right) \frac{l}{E} = \frac{\sigma_i l}{E}$
 $= \frac{79.58 \times 5 \times 10^3}{2 \times 10^5}$

$$\delta l = \underline{1.99 \text{ mm}}$$

(c) Strain energy absorbed $U = \frac{\sigma^2}{2E} \cdot A \cdot l$

$$= \frac{(79.58)^2}{2 \times 2 \times 10^5} \times 1256.63 \times 5 \times 10^3$$

$$U = \underline{19895.52 \text{ N-mm}}$$

prob 13 ! J.N.T.U Nov 2006 Suppl. NR 210102.

A crane chain having sectional area 650 mm^2 carries a load of 12 kN . As it is being lowered the chain got jammed suddenly, at which time the length of the chain unwound was 10 m . Calculate the stress induced in the chain due to sudden stoppage. Neglect the weight of the chain, take $E = 200 \text{ GPa}$.

Solution:

$$A = 650 \text{ mm}^2 ; P = 12 \text{ kN} ; l = 10 \text{ m}.$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

Stress due to sudden stoppage = ?

$$\text{Stress due to sudden loading } \sigma_s = \frac{2P}{A}$$

$$\sigma_s = \frac{2 \times 12 \times 10^3}{650}$$

$$\sigma_s = 36.92 \text{ N/mm}^2$$

Unit I Exercise Questions.

① Draw stress-strain relationship of mild steel and explain the important points.

② Write short notes on

(a) Hook's law

(b) working stress

(c) Factor of safety

(d) Lateral strain and poisson's ratio.

③ A steel wire is to support a load of 5 kN so that the tensile ~~stress~~ ^{stress} does not exceed 140 N/mm^2 .

Find (a) Minimum dia of wire required

(b) Extension of the wire if it is 3.5 m long. Take $E = 2.05 \times 10^5 \text{ N/mm}^2$.

Ans
 $\text{dia } d = 6.74 \text{ mm.}$

$\delta l = 2.77 \text{ mm.}$

④ A metal specimen 16 mm in dia is tested under a gauge length of 60 mm. At 22 kN the extension of was 0.04 mm. The maximum load taken by the specimen was 66 kN and the fracture occurred at 45 kN. Find

(a) Modulus of Elasticity 'E'.

(b) ultimate strength

(c) Breaking strength

(d) percentage elongation if the final length of specimen was 80 mm.

Ans
 $E = 1.64 \times 10^5 \text{ N/mm}^2$

ultimate strength
 $= 223.88 \text{ N/mm}^2$

Breaking strength
 $= 328.35 \text{ N/mm}^2$

percentage elongation
 $= 33.33\%$

- ⑤ A metallic bar $10\text{ mm} \times 10\text{ mm}$ cross-sectional area and 110 mm long is subjected to an axial force of 8 kN . The lateral dimensions of the bar was found to be changed to $9.998 \times 9.998\text{ mm}$. If modulus of rigidity of the material is $0.8 \times 10^5\text{ N/mm}^2$, determine poisson's ratio and modulus of elasticity if the bar elongates by 0.06 mm .

Ans

$$\text{poisson's ratio } \frac{1}{m} = 0.367$$

$$E = 2.18 \times 10^5\text{ N/mm}^2$$

- ⑥ A tapering rod measuring 40 mm at one end and 120 mm at the other measures 500 mm in length. The rod is subjected to an axial force 'P' which causes a change in length of 0.16 mm . Determine 'P' if $E = 2.05 \times 10^5\text{ N/mm}^2$ and thickness $t = 8\text{ mm}$.

Ans

$$P = 38.22\text{ kN}$$

- ⑦ A railway track was laid at 22°C , if the temperature is raised to 42°C . find

(a) Stress developed in the rails if no expansion is allowed.

(b) Change in length of rails if they are free to expand.

Take $\alpha_s = 11.9 \times 10^{-6}/^\circ\text{C}$ and $E_s = 2.05 \times 10^5\text{ N/mm}^2$
 Length of rail = 18 m

Ans

(a) 48.79 N/mm^2

(b) 4.28 mm

⑧ Show that the strain energy stored per unit volume u' , in a strained member, is given by the expression $\frac{\sigma^2}{2E}$, where σ is the stress in the member and E is the modulus of elasticity of material.

⑨ prove that stress due to suddenly applied load is twice the stress due to gradually applied load.

⑩ A square bar 40mm x 40mm cross-section and 5.5m long is subjected to axial tensile load of 60 kN. Determine

(a) Maximum instantaneous stress induced.

(b) Instantaneous elongation in the rod.

(c) Strain energy absorbed in the rod.

Take $E = 2.05 \times 10^5 \text{ N/mm}^2$

Ans

$$(a) \sigma_s = \frac{2P}{A} = 75 \text{ N/mm}^2$$

$$(b) \delta l = \frac{\sigma_s l}{E} = 2.01 \text{ mm}$$

$$(c) u = \frac{\sigma^2}{2E} \times Al = 120731.71 \text{ N-mm.}$$

(v) Continuous Beam:-

A beam which is provided with more than two supports as shown is known as continuous beams.



2.1 Definition of Beam:

A beam may be defined as a horizontal bar subjected to vertical loads.

These vertical loads are also called as transverse loads as they act perpendicular to the axis of the beam.

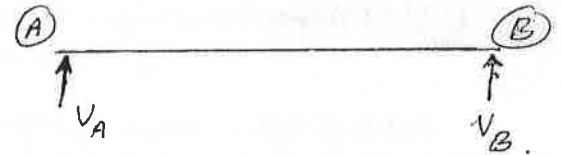
~~Depending on~~

2.2 Types of Beams:

Depending upon the end conditions, the beams are classified as

(i) Simply Supported Beams

It has only vertical reaction V
→ Does not offer horizontal reaction and moment.



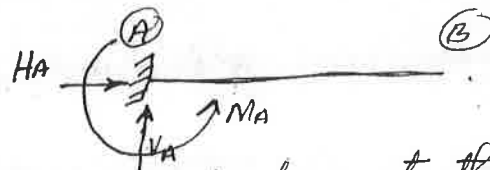
(ii) Hinged beam

→ It offers vertical reaction V and horizontal reaction H
→ Does not offer resistance to moment



(iii) Cantilever Beam

It is fixed at one end and free at the other
→ It offers vertical reaction V , Horizontal reaction H and ^{resisting} moment M at fixed end.



(iv) Overhanging Beam.

It is same as simply supported beam with projected beam on both ends or one end.

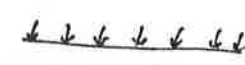
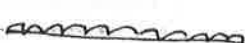


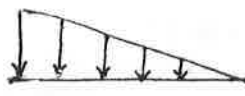
→ All the above beams are statically determinate i.e. they can be analysed by equilibrium equations.

→ Equilibrium equations are $\sum H = 0$ (or $\sum X = 0$)
 $\sum V = 0$ (or $\sum Y = 0$)
 $\sum M = 0$ (or $\sum M_z = 0$)

2.3 Types of Loads:

(i) point load or concentrated load ↓

(ii) uniformly distributed load (udl). 
or 

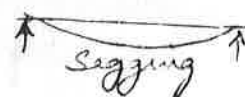
(iii) uniformly varying load  

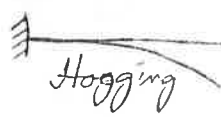
SHEAR FORCE (↓)

It is the net vertical force on left hand side or right hand side of a given section. (↓)

Bending Moment (⇌)

It is the algebraic sum of moment on left hand side or right hand side of a given section.

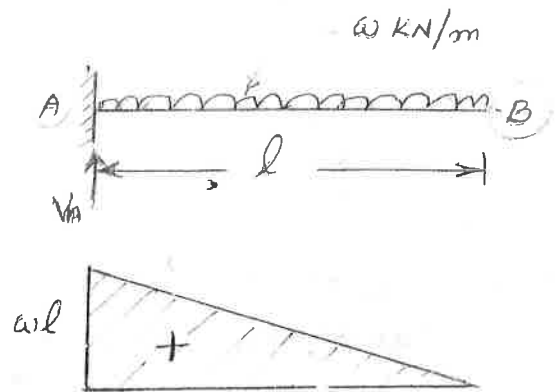
→ Sagging moment is taken as POSITIVE 

→ Hogging moment is taken as NEGATIVE 

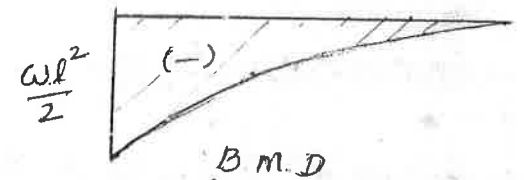
prob: 1
 A cantilever beam of span l is subjected to udl of intensity ' w '. Draw Shear force and Bending moment diagrams.

SOLUTION:

Support reaction
 at 'A' $V_A = w \cdot l$.



S.F.D
 $\rightarrow$ SF at 'A' $SF_A = V_A = w \cdot l$.
 $\rightarrow$ SF at distance ' x ' from 'A'



$$SF_x = V_A - w \cdot x$$

$$= w \cdot l - w \cdot x$$

$\rightarrow$ SF at B (i.e. $x = l$).

$$SF_B = w \cdot l - w \cdot l$$

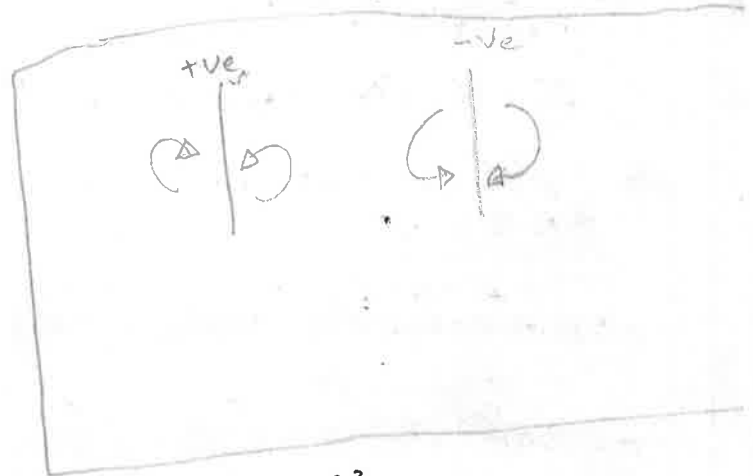
$$= \underline{\underline{0}}$$

B.M.D

$\rightarrow$ BM at 'A' $M_A = w \cdot l \times \frac{l}{2} = \frac{w \cdot l^2}{2}$.

$\rightarrow$ BM at B $M_B = 0$.

Note: Since the equation of BM is square of distance therefore BMD is a curve.



Prob: 2:

Q. 4)

A cantilever beam of span 'l' is subjected to a concentrated load 'W' at the free end. Draw SFD and BM diagram.

Solution:

→ Support reaction at A

Sum of vertical forces $\Sigma V = 0$

$$V_A - W = 0 \quad \text{or} \quad \boxed{V_A = W \text{ KN}}$$

SFD

→ SF at 'A' $V_A = W$

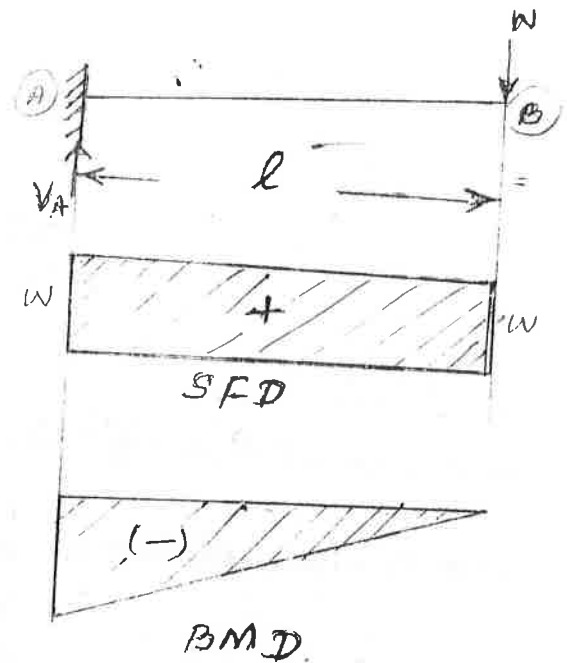
→ SF at 'B' $V_B = V_A - W$
 $= W - W$
 $= 0$

BMD

→ BM at 'A' $M_A = W \times l$

→ BM at B $M_B = 0$

NOTE: Since BM equation is to the single power of distance 'l' therefore BMD is a straight line.



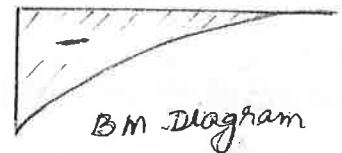
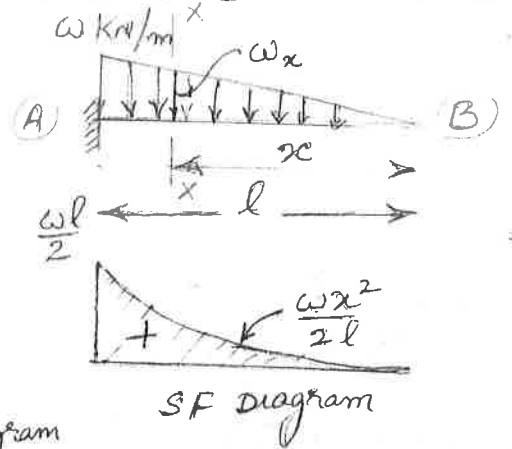
Prob 3.
A cantilever beam of span 'l' carries a uniformly varying load whose intensity varies from zero at free end to 'w' at the fixed end. Draw SF and BM diagrams.

SOLUTION:

Intensity of load at free end = 0

Intensity of load at fixed end = w.

Support reaction at 'A' $\left\{ \begin{aligned} V_A &= \text{Area loading diagram} \\ &= \frac{1}{2} \times l \times w \\ V_A &= \frac{1}{2} wl. \end{aligned} \right.$



Intensity of load w_x at a distance 'x' from free end 'B'

From similar triangles $\frac{w_x}{w} = \frac{x}{l} \Rightarrow \boxed{w_x = w \cdot \frac{x}{l}}$

SFD

→ SF at A $SF_A = V_A = \frac{wl}{2}$

→ SF at distance $(l-x)$ from A
i.e. at distance 'x' from B $\left\{ \begin{aligned} SF_{x-x} &= V_A - \text{Load between A and section x-x} \end{aligned} \right.$

$SF_{x-x} = \frac{wl}{2} - \left(\frac{w + w_x}{2} \right) (l-x)$

$= \frac{wl}{2} - \frac{w + \frac{w \cdot x}{l}}{2} (l-x)$

$= \frac{wl}{2} - \frac{wl + wx}{2l} (l-x)$

$= \frac{wl}{2} - \frac{wl^2}{2l} + \frac{wx^2}{2l}$

$= \frac{wl}{2} - \frac{wl}{2} + \frac{wx^2}{2l}$

$\boxed{SF_{x-x} = -\frac{wx^2}{2l}}$

→ SF at 'A' $SF_A = V_A = \frac{\omega l}{2}$

→ SF at section x-x $SF_{x-x} = \text{Load between x-x and B}$
 $= \frac{1}{2} x \cdot \frac{\omega}{l} x$

$$SF_{x-x} = \frac{\omega x^2}{2l}$$

→ SF at B. $SF_B = V_A - \text{Load between A \& B.}$
 $= \frac{\omega l}{2} - \frac{\omega l}{2}$

$$SF_B = 0$$

BMD

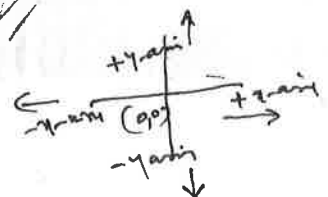
→ BM at section x-x $M_x = \text{Load on right of x-x} \times \text{distance of load from x-x}$

$$M_x = \left(\frac{1}{2} \frac{\omega x}{l} x \right) \times \frac{x}{3}$$

$$M_x = \frac{\omega x^3}{6l} \quad \text{Hogging.}$$

→ BM at 'A' i.e. (x=l) $M_A = \frac{\omega l^3}{6l}$

$$M_A = \frac{\omega l^2}{6}$$



(↑) upward forces (+ve)

(↓) downward force (-ve)

Note → In Shear force length is not considered only vertical loads.

* Prob 4 A.C.W Right $\rightarrow$ " " (-) " " (+)
 A simply supported beam (ss beam) of span 'l' is subjected to a central point load of intensity 'W'. Draw SF and BM diagram.

SOLUTION:

$\rightarrow$ To find vertical reactions

V_A and V_B .

$\rightarrow$ Sum of vertical forces $\Sigma V = 0$

$$V_A + V_B - W = 0$$

$$V_A + V_B = W \quad \text{--- (1)}$$

$\rightarrow$ Taking moment about 'B', $\Sigma M_B = 0$

$$(V_A \times l) - W \times \frac{l}{2} = 0$$

$$\boxed{V_A = \frac{W}{2}}$$

Substituting V_A in Eqn (1) $\frac{W}{2} + V_B = W$

$$\boxed{V_B = \frac{W}{2}}$$

SFD

$\rightarrow$ SF at A $SF_A = V_A = \frac{W}{2}$

SF at C $SF_C = \frac{W}{2} - W = -\frac{W}{2}$

$\rightarrow$ SF at B $SF_B = -\frac{W}{2} + V_B = -\frac{W}{2} + \frac{W}{2} = 0$

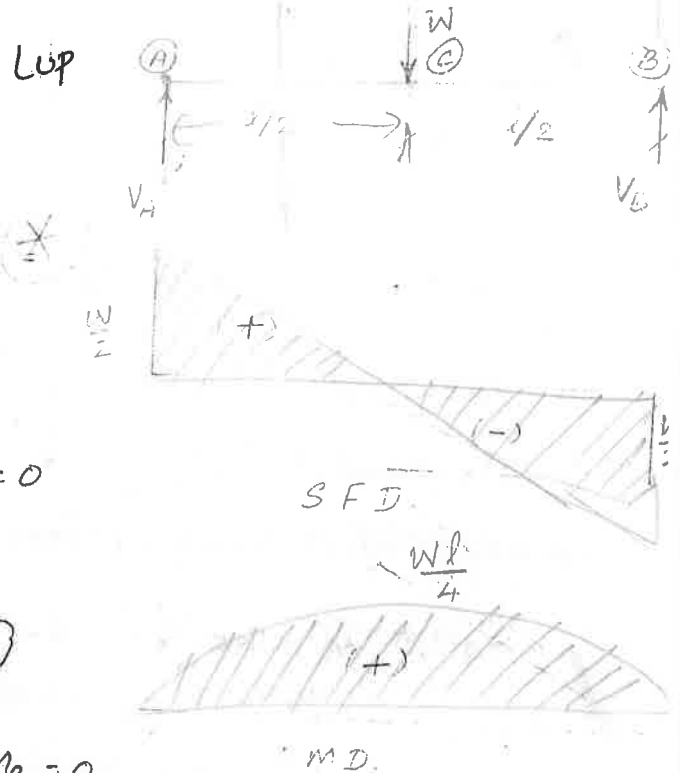
BMD

BM at A $M_A =$ BM at B $M_B = 0$

BM at C $M_C = V_A \times \frac{l}{2} = \frac{W}{2} \times \frac{l}{2}$

$$\boxed{M_C = \frac{Wl}{4}}$$

Since BM equation is to the single power of distance therefore BMD is STRAIGHT LINE



Prob no: 5

* A simply supported beam of span 'l' is subjected to a uniformly distributed load (udl) w kN/m on entire span. Draw SFD and BMD.

SOLUTION:

To find support reactions

V_A and V_B .

→ Sum of vertical forces $\Sigma V = 0$

$$V_A + V_B - wl = 0$$

$$V_A + V_B = wl \quad \text{--- (1)}$$

→ Taking moment about B $\Sigma M_B = 0$

$$V_A \times l - wl \times \frac{l}{2} = 0$$

$$V_A = \frac{wl}{2}$$

Substitute V_A in Eqn (1). $\frac{wl}{2} + V_B = wl$

$$V_B = \frac{wl}{2}$$

SHEAR FORCE DIAGRAM

→ SF at A = $V_A = \frac{wl}{2}$

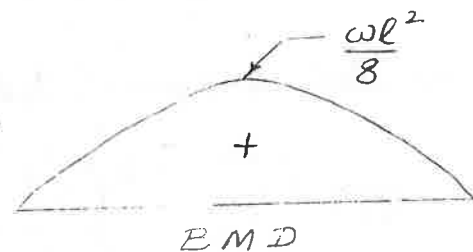
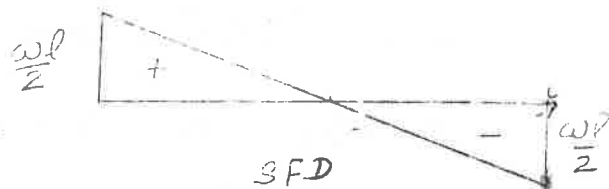
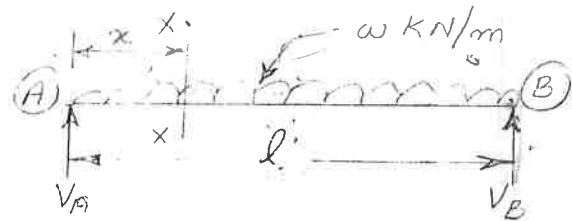
→ SF at distance 'x' from A $SF_x = V_A - w \cdot x$

→ SF at Centre ($x = \frac{l}{2}$) $SF_{x=\frac{l}{2}} = \frac{wl}{2} - \frac{wl}{2} = 0$

→ SF at just left of B. $SF_{B-left} = SF_x - w \cdot \frac{l}{2}$
 $= 0 - \frac{wl}{2}$

$$SF_{B-left} = -\frac{wl}{2}$$

→ SF at just right of B $SF_{B-right} = -\frac{wl}{2} + V_B$



$$\Rightarrow \sum B_{-right} = -\frac{\omega l}{2} + \frac{\omega l}{2}$$

$$= \underline{\underline{0}}$$

Bending Moment Diagram:

BM at A M_A = BM at B M_B = 0.

$$\begin{aligned} \text{BM at distance 'x' from 'A' } M_x &= V_A \cdot x - \omega \cdot x \cdot \frac{x}{2} \\ &= \frac{\omega l}{2} \cdot x - \frac{\omega x^2}{2} \end{aligned}$$

$$\begin{aligned} \text{BM at centre } (x = \frac{l}{2}) M_c &= \frac{\omega l}{2} \cdot \frac{l}{2} - \frac{\omega}{2} \left(\frac{l}{2}\right)^2 \\ &= \frac{\omega l^2}{4} - \frac{\omega l^2}{8} \end{aligned}$$

$$M_c = \frac{\omega l^2}{8}$$

$\therefore$ The BM equation is to the second power of distance, therefore BMD is a curve.

OVERHANGING BEAMS.

Prob 6:

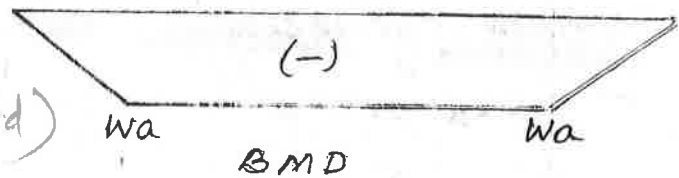
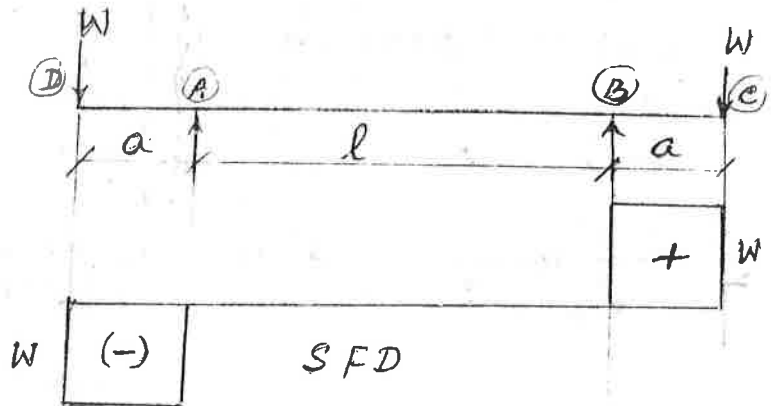
The supports of an overhanging beam are spaced at 'l'm and it has overhang of 'a'm on either side. The beam is subjected to concentrated loads 'W' at the free ends. Draw SF and BMD.

SOLUTION

Let V_A and V_B be the reactions at supports A and B.

Since the loading is symmetrical, therefore

$$V_A = V_B = W \quad \text{! (Total load)}$$



SFD

$$\rightarrow \text{SF at D} = -W$$

$$\text{SF at left of A} = -W$$

$$\text{SF at just right of A} = (\text{SF}_{A-\text{left}} + V_A) = (-W + W) = \underline{\underline{0}}$$

$$\text{SF at just right of B} = (\text{SF}_{B-\text{left}} + V_B) = (0 + W) = \underline{\underline{W}}$$

$$\text{SF at C} = \text{SF}_{B-\text{right}} - W = W - W = 0.$$

BMD

BMD

→ BM at D = BM at C = 0

→ BM at A = $-wa$.

→ BM at dist 'x' from D.

$$\begin{aligned} BM_x &= -W \cdot x + V_A (x-a) \\ &= -Wx + W(x-a) \\ &= -Wx + Wx - wa \end{aligned}$$

$$BM_x = -wa$$

→ BM at B (Considering right side of B)

$$BM_B = -W \cdot a \text{ (Hogging i.e. -ve)}$$

Prob 4:

A simply supported beam with equal overhangs of length 'a' and simply supported span 'l' is subjected to a udl 'w' on the entire span. Draw SF and BM diagrams.

SOLUTION:

From Symmetry $V_A = V_B$

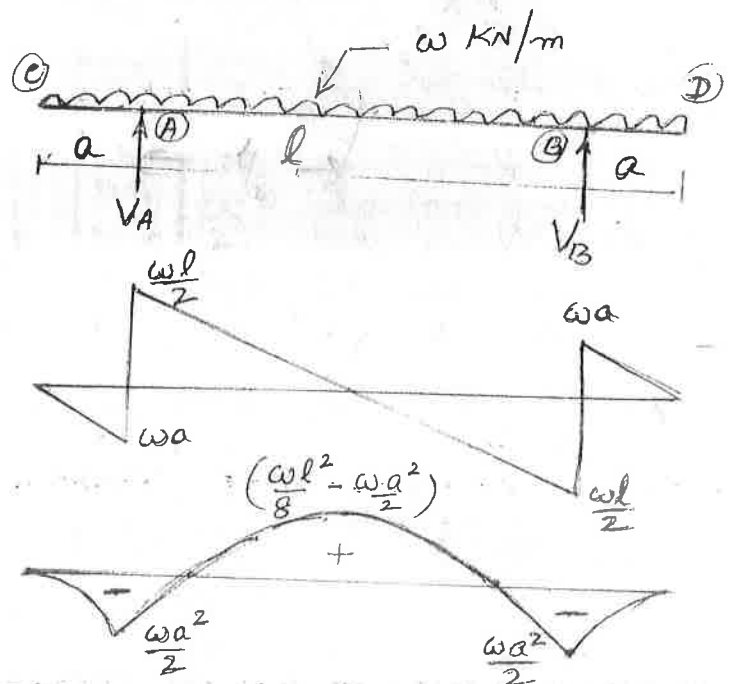
$$V_A = V_B = \frac{w(l+2a)}{2}$$

SFD

→ SF at C = 0

→ SF at left of 'A' = $-w \cdot a$

$$SF_{A-left} = -wa$$



$$\begin{aligned} \rightarrow \text{SF at right of 'A'} \quad SF_A &= -wa + V_A \\ &= -wa + \frac{\omega(l+2a)}{2} \\ &= -wa + \frac{\omega l}{2} + wa \end{aligned}$$

$$\boxed{SF_{A-\text{right}} = \frac{\omega l}{2}}$$

$$\begin{aligned} \rightarrow \text{SF at left of 'B'} \quad SF_{B-\text{left}} &= SF_A - \omega l \\ &= \frac{\omega l}{2} - \omega l \end{aligned}$$

$$\boxed{SF_{B-\text{left}} = -\frac{\omega l}{2}}$$

$$\begin{aligned} \rightarrow \text{SF at right of 'B'} \quad SF_{B-\text{right}} &= SF_{B-\text{left}} + V_B \\ &= -\frac{\omega l}{2} + \frac{\omega(l+2a)}{2} \\ &= -\frac{\omega l}{2} + \frac{\omega l}{2} + wa \end{aligned}$$

$$\boxed{SF_{B-\text{right}} = wa}$$

$$\rightarrow \text{SF at D} \quad SF_D = wa - \omega a$$

$$\boxed{SF_D = 0}$$

B.M.D.

$$\rightarrow \text{BM at C} = \text{BM at D} = 0.$$

$$\rightarrow \text{BM at A} \quad M_A = \text{BM at B} \quad M_B = -\omega a \times \frac{a}{2} = -\frac{\omega a^2}{2}$$

$$\rightarrow \text{BM at Centre of beam} \quad M_{\text{Centre}} = V_A \times \frac{l}{2} - \omega \left(\frac{l}{2} + a\right) \left(\frac{l}{2} + a\right)$$

$$M_{\text{Centre}} = \frac{\omega(l+2a)}{2} \times \frac{l}{2} - \frac{\omega}{2} \left(\frac{l^2}{4} + a^2 + la\right)$$

$$= \frac{\omega l^2}{4} + \frac{\omega a l}{2} - \frac{\omega l^2}{8} - \frac{\omega a^2}{2} - \frac{\omega a l}{2}$$

A cantilever of length 5.0 m is loaded as shown in figure below. Draw SF and BM diagram for the cantilever.

Solution:

Support reaction at A V_A
Assume V_A acting upward.
 $\sum V = 0$.

$$V_A - 0.25 - 1 \times 2 + 2.5 = 0$$

$V_A = -0.25 \text{ KN}$ $\therefore$ The value is -ve $\therefore V_A$ acts downward.

SFD

$$\rightarrow \text{SF at A } SF_A = V_A = -0.25 \text{ KN}$$

$$\rightarrow \text{SF at right of E } SF_E = -V_A - 250 \text{ N}$$

$$SF_E = -0.25 - 0.25$$

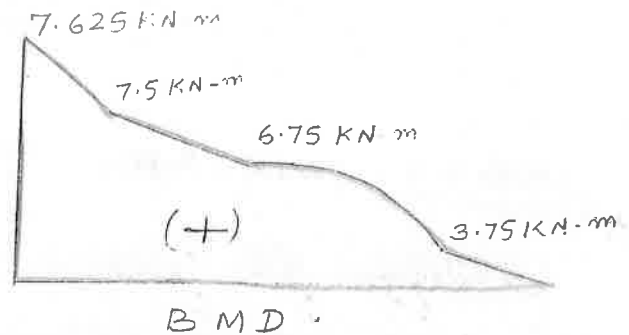
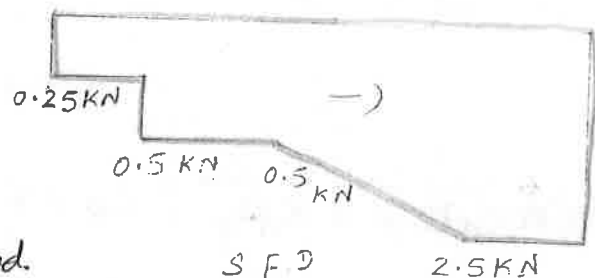
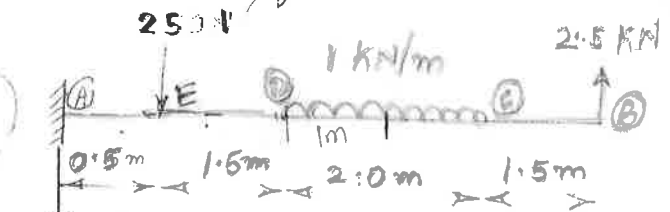
$$= -0.50 \text{ KN}$$

$$\rightarrow \text{SF at left of D } SF_{D-\text{left}} = -0.50 \text{ KN}$$

$$\rightarrow \text{SF at C } = -0.50 - 1 \times 2 = -2.5 \text{ KN}$$

$$\rightarrow \text{SF at B } SF_B = -2.5 + 2.5 \text{ KN}$$

$$= 0$$



(14)
BMD starting from free end.

BM at B $M_B = 0$.

BM at C $M_C = 2.5 \times 1.5 = \underline{3.75} \text{ KN-m. (straight line, and Sagging +ve)}$

BM at D $M_D = 2.5 \times 3.5 - 1 \times 2 \times \frac{2}{2} = \underline{6.75} \text{ KN-m (Sagging +ve)}$

BM at E $M_E = 2.5 \times 5 - 1 \times 2 \times (1.5 + 1) = 7.5 \text{ KN-m. (Sagging +ve)}$

BM at A $M_A = 2.5 \times 5.5 - 1 \times 2 \times 3 - 0.25 \times 0.5 = \underline{7.625} \text{ KN-m. (Sagging +ve)}$

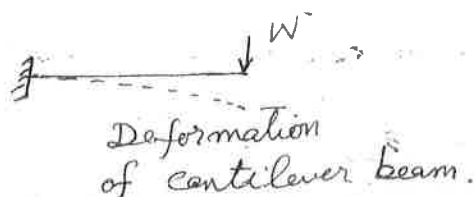
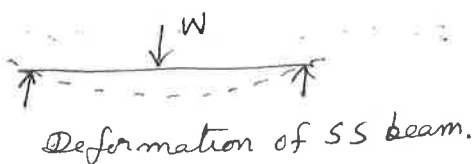
Prob 8: JNTU Supp. May/June 2009 Code NO RR 210102.

Q Define the "Beam" and the type of action and deformation it undergoes.

Ans: A beam may be defined as a horizontal long member subjected to vertical loads.

Beams are classified as (a) Cantilever beam (b) Simply supported beam (c) Overhanging beam etc.

A beam deflects downward under applied load.



(D) Draw SF and BM diagrams for a simply supported beam of span L m loaded with UDL of w KN/m.

Ans. Refer Prob NO: 5.

PROB 9: JNTU Supp 2009 NR Code NO NR 210102.

A simply supported beam with overhanging ends carries transverse loads as shown. If $wL = P$, what is the ratio of a/L for which BM at the middle of the beam will be zero.

SOLUTION

Since the loading is symmetrical

$$\therefore V_A = V_B = \frac{2P + wL}{2}$$

→ BM at middle of beam M_c

$$\begin{aligned} M_c &= V_A \times \frac{L}{2} - P\left(\frac{L}{2} + a\right) - w \cdot \frac{L}{2} \times \frac{L}{4} \\ &= \frac{2P + wL}{2} \times \frac{L}{2} - \frac{PL}{2} - P \cdot a - \frac{wL^2}{8} \end{aligned}$$

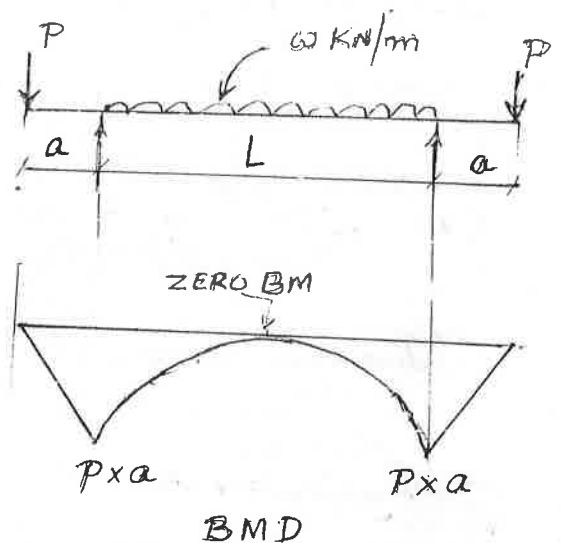
$$= \frac{2PL}{4} + \frac{wL^2}{2} - \frac{PL}{2} - Pa - \frac{wL^2}{8}$$

Substituting $P = wL$ and BM $M_c = 0$.

$$\begin{aligned} 0 &= \frac{2wL \cdot L}{4} + \frac{wL^2}{2} - \frac{wL \cdot L}{2} - \underline{wL \cdot a} - \frac{wL^2}{8} \\ &= \frac{wL^2}{2} + \frac{wL^2}{2} - \frac{wL^2}{2} - \underline{wL \cdot a} - \frac{wL^2}{8} \end{aligned}$$

$$wL \cdot a = \frac{wL^2}{2} - \frac{wL^2}{8}$$

$$\boxed{a = \frac{3L}{8}}$$



Prob 101 JNTU May/June 2009 Supp. Set No 2 Code No 43001.

A simply supported beam of length 6m, carries point load of 3kN and 4kN at distances of 2m and 4m from the left end. Draw the shear force and bending moment diagrams for the beam.

SOLUTION:

To find Support reactions.

$$\sum V = 0$$

$$V_A + V_B - 3 - 4 = 0$$

$$V_A + V_B = 7 \quad \text{--- (1)}$$

Taking moment about B.

$$\sum M_B = 0$$

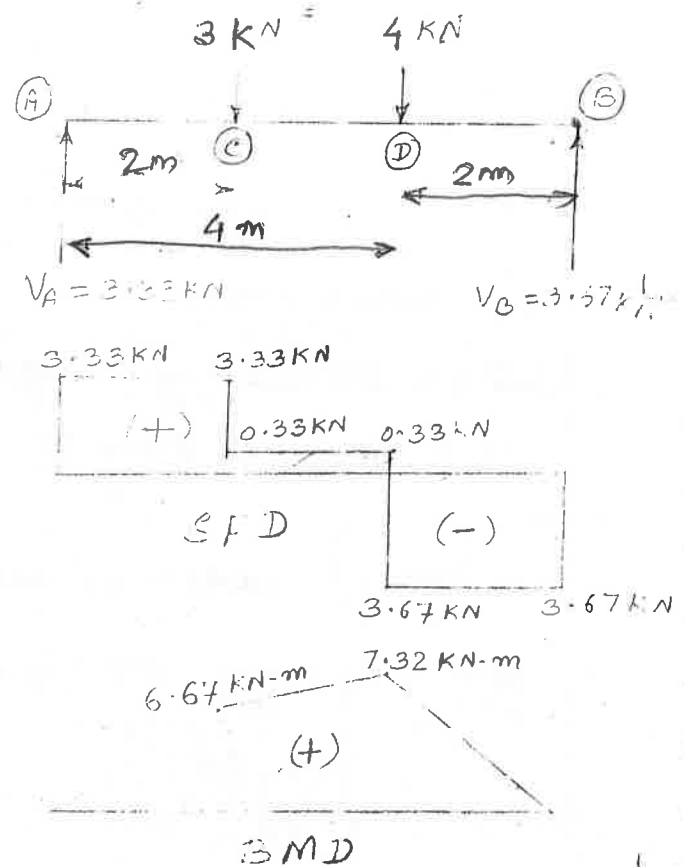
$$V_A \times 6 - 4 \times 2 - 3 \times 4 = 0$$

$$V_A = \frac{20}{6} = \underline{3.33 \text{ kN}}$$

Substituting V_A in Eqn (1)

$$V_B = 7 - 3.33$$

$$\boxed{V_B = 3.67 \text{ kN}}$$



SFD

$$\rightarrow \text{SF at A } SF_A = V_A = 3.33 \text{ kN}$$

$$\rightarrow \text{SF at left of C } SF_{C-\text{left}} = 3.33 \text{ kN}$$

$$\rightarrow \text{SF at right of C } SF_{C-\text{right}} = 3.33 - 3 = \underline{0.33 \text{ kN}}$$

$$\rightarrow \text{SF at left of D } SF_{D-\text{left}} = 0.33 \text{ kN}$$

$$\rightarrow \text{SF at right of D } SF_{D-\text{right}} = (0.33 - 4) = -3.67 \text{ kN}$$

$$\rightarrow \text{SF at B } SF_B = -3.67 + V_B = (-3.67 + 3.67)$$

$$BM \text{ at } A = BM \text{ at } B = 0$$

$$BM \text{ at } C \quad M_c = V_A \times 2$$

$$= 3.33 \times 2$$

$$M_c = 6.66 \text{ KN-m}$$

$$BM \text{ at } D \quad M_D = V_A \times 4 - 3 \times 2$$

$$= 3.33 \times 4 - 3 \times 2$$

$$= \underline{7.32 \text{ KN-m}}$$

prob 11:

Draw shear force and Bending moment diagram for a simply supported beam of length 8m and carrying a uniformly distributed load of 12 KN/m for a distance of 4m from the left end. Also calculate the maximum BM on the section.

SOLUTION:

To find support reactions V_A & V_B .

$$\sum V = 0$$

$$\rightarrow V_A + V_B - 12 \times 4 = 0$$

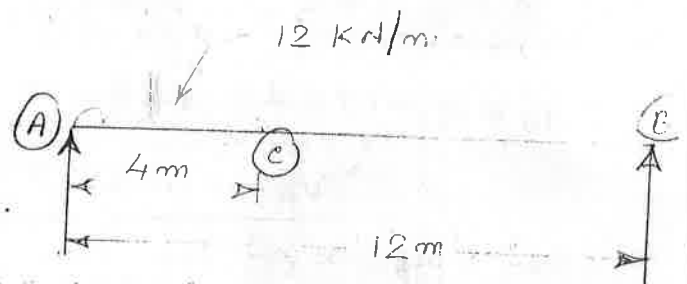
$$V_A + V_B = 48 \quad \text{--- (1)}$$

$$\rightarrow \sum M_B = 0$$

$$V_A \times 12 - 12 \times 4 \times 10 = 0$$

$$V_A = 40 \text{ KN}$$

Substituting in (1) $V_B = 8 \text{ KN}$



23/10/20

Prob 11: Draw shear force and Bending moment diagram

Draw shear force and bending moment diagram for a simply supported beam of ~~8m~~ length 8m and carrying a uniformly distributed load of 12 kN/m for a distance of 4m from the left end. Also calculate the maximum bending moment on the section.

SOLUTIONS:

To find support reactions V_A & V_B

$\Sigma V = 0$

$V_A + V_B - 12 \times 4 = 0$

$V_A + V_B = 48$ — (1)

$\Sigma M_B = 0$

$V_A \times 8 - 12 \times 4 \times \frac{4}{2} = 0$

$V_A = 36 \text{ kN}$ ✓

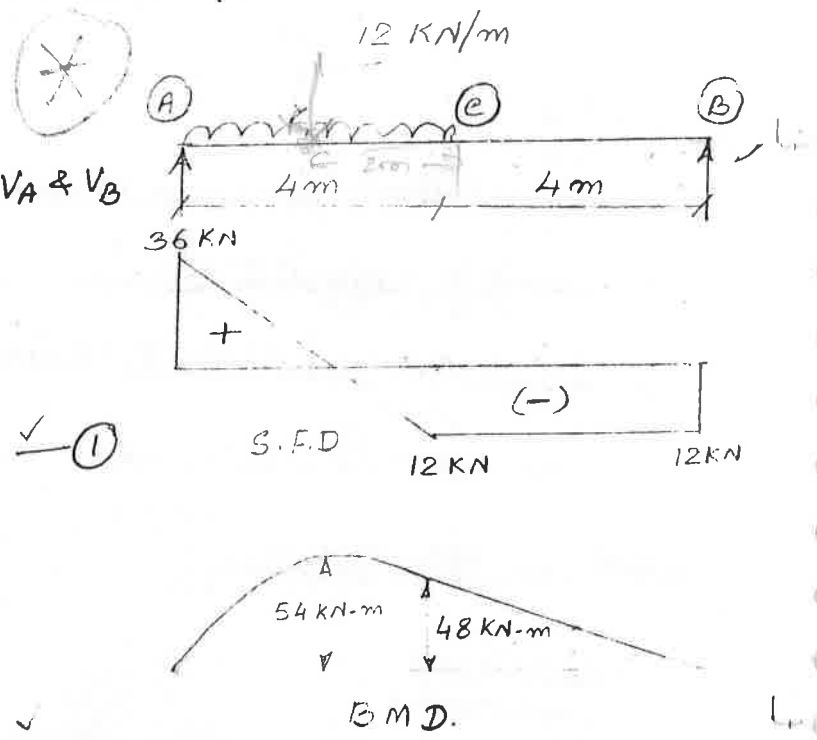
Substituting in (1) $V_B = 12 \text{ kN}$

S.F.D

→ SF at A $SF_A = V_A = 36 \text{ kN}$.

→ SF at C $SF_C = V_A - 12 \times 4 = 36 - 48 = -12 \text{ kN}$

→ SF at B $SF_B = SF_C + V_B = -12 + 12 = 0$.



BMD

BM at A, $M_A = \text{BM at B, } M_B = 0$.

BM at distance ' x ' from A (Between A and C).

$$M_x = V_A \cdot x - 12 \times x \times \frac{x}{2} \\ = 36x - 6x^2.$$

BM at $x = 4 \text{ m}$.

$$M_{x=4} = 36 \times 4 - 6 \times 4^2 \\ = \underline{48 \text{ KN-m.}} \quad \begin{array}{l} \text{(parabolic from A to C)} \\ \text{straight line from C to B.} \end{array}$$

Max. BM

$$\rightarrow M_x = 36x - 6x^2$$

$$\text{For maximum BM } \frac{dM_x}{dx} = 0$$

$$\frac{dM_x}{dx} = 36 - 12x.$$

$$0 = 36 - 12x.$$

$$\boxed{x = 3 \text{ m}}$$

$$\rightarrow M_{\text{max}} = 36 \times 3 - 6 \times 3^2 \\ = \underline{54 \text{ KN-m}} \checkmark$$

NOTE: BM will be maximum where SF changes its sign.

(20)

Prob 12: JNTU Supp Feb 2007 NR code No NR 210102.

Construct S.F.D and B.M.D for the cantilever beam shown in figure and identify the maximum value for each.

SOLUTION:

To find support reaction V_A

$\Sigma V = 0$

$$V_A - 10 \times 2 - 5 = 0$$

$V_A = 25 \text{ KN}$

S.F.D

$$\rightarrow \text{SF at A} = V_A = 25 \text{ KN.}$$

$$\begin{aligned} \rightarrow \text{SF at C } SF_C &= V_A - 10 \times 2 \\ &= 25 - 20 \\ &= \underline{5 \text{ KN.}} \end{aligned}$$

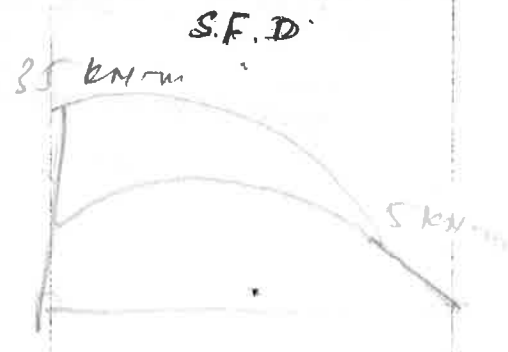
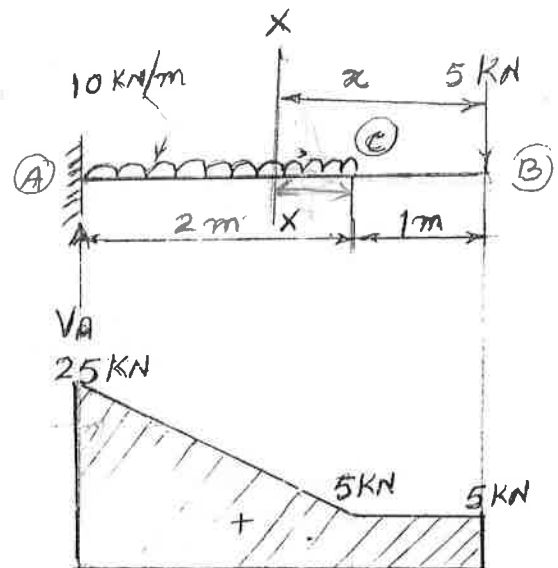
$$\begin{aligned} \rightarrow \text{SF at B } SF_B &= SF_C - 5 \\ &= 5 - 5 \\ &= \underline{\underline{0}} \end{aligned}$$

B.M.D

$$\rightarrow \text{BM at B } M_B = 0$$

$$\rightarrow \text{BM at C } M_C = 5 \times 1 = 5 \text{ KN-m (Hogging -ve)}$$

~~$\rightarrow$ BM between~~



→ Consider a section $x-x$ at a distance ' x ' from free end B.

$$\begin{aligned}\rightarrow \text{BM at } x-x \quad M_x &= 5x + 10(x-1) \frac{(x-1)}{2} \\ &= 5x + 5(x^2 + 1 - 2x) \\ &= 5x + 5x^2 + 5 - 10x \\ &= \underline{5x^2 - 5x + 5}.\end{aligned}$$

→ BM at A M_A (i.e. at $x = 3\text{ m}$)

$$\begin{aligned}M_A &= 5 \times 3^2 - 5 \times 3 + 5 \\ &= 45 - 15 + 5 \\ &= \underline{35} \text{ kN-m}.\end{aligned}$$

→ Max BM:

Max BM occurs

Prob 13: JNTU Regular NOV 2007 Set No 1. Code R050210101.

Construct S.F.D and B.M.D for the beam with overhangs shown in figure.

SOLUTION

To find vertical reactions V_A and V_B .

→ $\sum V = 0$ (Sum of Vertical forces)

$$V_A + V_B - 20 - 20 - 20 \times 4 = 0$$

$$V_A + V_B = 120 \quad \text{--- (1)}$$

→ $\sum M_B = 0$ (Sum of moments at B)

$$V_A \times 4 + (20 \times 2) - (20 \times 6) - 20 \times 4 \times \frac{4}{2} = 0$$

$$4V_A + 40 - 120 - 160 = 0$$

$$\boxed{V_A = 60 \text{ KN}}$$

→ Substituting V_A in Eqn (1) $\boxed{V_B = 60 \text{ KN}}$

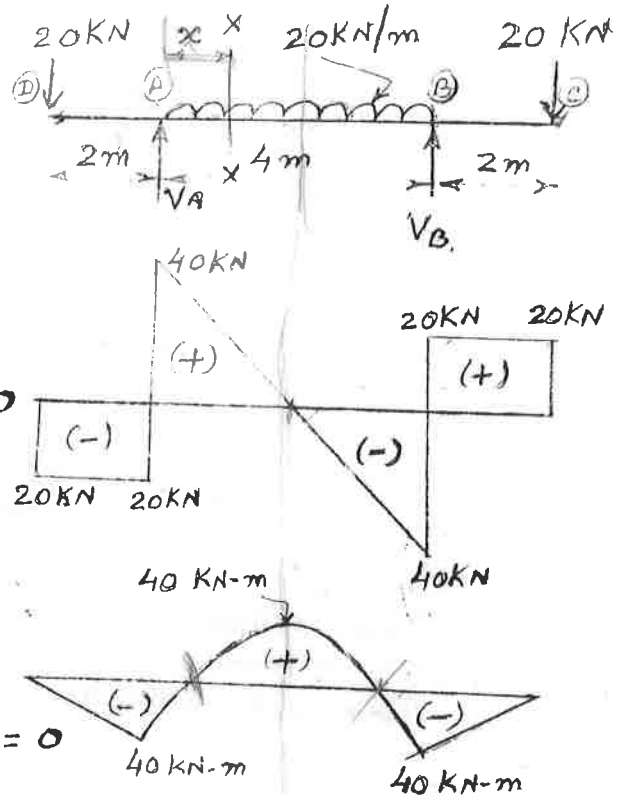
S.F.D

→ SF at D $SF_D = -20 \text{ KN}$.

→ SF at left of A $SF_{A-\text{left}} = -20 \text{ KN}$.

→ SF at right of A $SF_{A-\text{right}} = -20 + 60 = 40 \text{ KN}$.

→ SF at left of B $SF_{B-\text{left}} = 40 - 20 \times 4 = -40 \text{ KN}$.



$$q = \frac{36}{bh^3} \times \frac{h}{b \cdot y} \times \frac{1}{3} \times \frac{b y^2}{h} (h - y)$$

$$q = \frac{12 SF}{bh^3} y (h - y) \quad \text{--- (1)}$$

For q to be maximum $\frac{dq}{dy} = 0$.

Differentiating above equation

$$\frac{dq}{dy} = \frac{12 SF}{bh^3} (h - 2y) = 0$$

$$h = 2y$$

$$\text{or } y = \frac{h}{2}$$

Substitute $y = \frac{h}{2}$ in Eqn (1).

$$q_{\max} = \frac{12 SF}{bh^3} \cdot \frac{h}{2} \left(h - \frac{h}{2} \right)$$

$$= \frac{12 SF}{bh^3} \times \frac{h}{2} \times \frac{h}{2}$$

$$q_{\max} = \frac{3 SF}{bh}$$

→ Shear stress at NA i.e. $y = \frac{2}{3}h$; Substitute in (1)

$$q_{NA} = \frac{12 SF}{bh^3} \times \frac{2}{3}h \left(h - \frac{2}{3}h \right)$$

$$= \frac{8 SF}{bh^2} \left(\frac{h}{3} \right)$$

$$q_{NA} = \frac{8 SF}{3 bh}$$

Prob 1: JNTU May/June 2009 Suppl. Set No ① Code 43001.

- (a) Prove that the shear stress 'q' at any point (or in a fibre) in the cross-section of a beam which is subjected to a shear force F, is given by

$$q = \frac{F}{I b} (\bar{A} \bar{y})$$

Solution:

Refer page no ①, ② & ③ for proof.

- (b) A beam of triangular section having base width 20 cm and height 30 cm is subjected to a shear force of 3 kN. Find the value of maximum shear stress and sketch the shear stress distribution along the depth of beam.

Solution:

$$b = 20 \text{ cm} = 200 \text{ mm} ; h = 30 \text{ cm} = 300 \text{ mm}.$$

$$F = SF = 3 \text{ kN} = \underline{\underline{3 \times 10^3 \text{ N}}}$$

$$\text{Shear stress } q = \frac{SF}{I b} (\bar{A} \bar{y}).$$

For triangular section.

$$q_{\text{max}} = 3 \times \frac{SF}{b h}.$$

$$q_{\text{max}} = 3 \times \frac{3 \times 10^3}{200 \times 300}$$

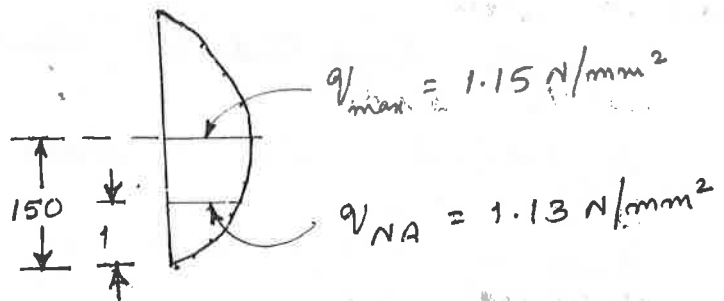
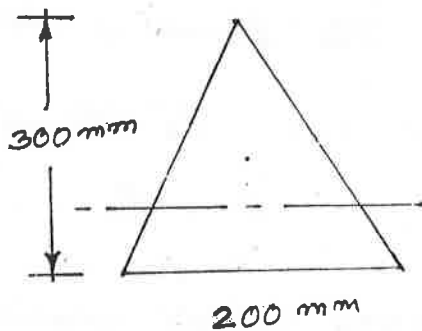
$$q_{\text{max}} = 0.15 \text{ N/mm}^2$$

→ Shear stress at NA.

For triangular section $q_{NA} = \frac{8}{3} \frac{SF}{bh}$

$$= \frac{8}{3} \times \frac{3 \times 10^3}{200 \times 300}$$

$$q_{NA} = 0.13 \text{ N/mm}^2$$



$$\frac{f}{R} = \frac{M}{I} = \frac{f}{Y} = \frac{f}{R}$$

$$\frac{fAY}{I \cdot b}$$

Prob 2: JNTU May/June 2009. Suppl. Set No (2)

(a) prove that the shear stress 'q' distribution in a rectangular section of a beam $b \times d$ which is subjected to a shear force 'F' is given by.

$$q = \frac{F}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

Solution:

Refer Page No (4) for derivation

(b) A wooden beam 10cm wide and 15cm deep is to resist longitudinal shear. The beam is simply supported over a span of 2m. If the allowable shearing stress is 0.45 MN/m^2 . Find the concentrated load that the beam can carry at its centre.

Soln: $b = 10 \text{ cm} = 100 \text{ mm}$.

$d = 15 \text{ cm} = 150 \text{ mm}$.

$l = 2 \text{ m}$.

$q = 0.45 \text{ MN/m}^2 = 0.45 \times \frac{10^6}{10^3 \times 10^3} \text{ N/mm}^2$
 $= 0.45 \text{ N/mm}^2$.

We know $q = \frac{SF}{Ib} (A\bar{y})$

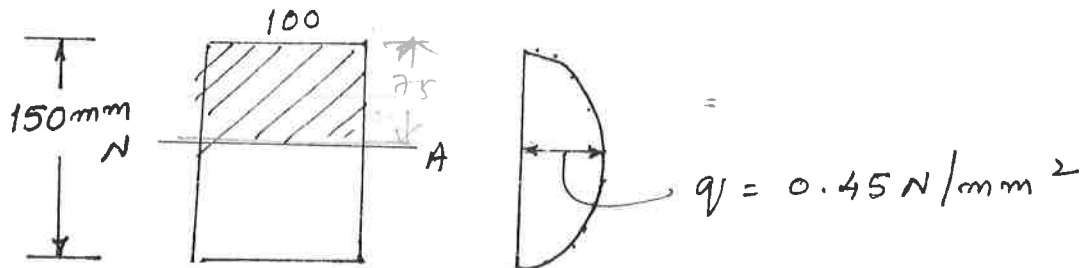
SF = ? To be calculated.

$I = \frac{bd^3}{12}$

$$I = \frac{100 \times 150^3}{12}$$

$$= 28.125 \times 10^6 \text{ mm}^4$$

Max Shear stress occur at NA.



$A\bar{y}$ = Moment of portion above the section considered about NA.

$$= (100 \times 75) \frac{75}{2}$$

$$= 281250 \text{ mm}^3$$

(dist of Centroidal axis of the shaded portion from neutral axis)

Substituting in standard equation

$$q = \frac{SF}{Ib} (A\bar{y})$$

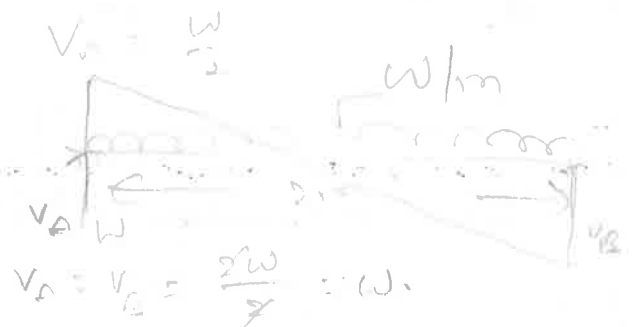
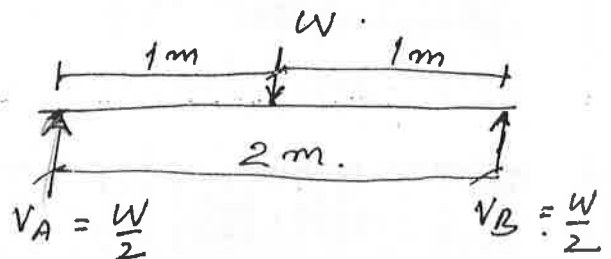
$$0.45 = \frac{SF}{28.125 \times 10^6} \times 281250$$

$$SF = 45 \text{ N}$$

$$SF = V_A = \frac{W}{2}$$

$$45 = \frac{W}{2}$$

$$W = 90 \text{ N}$$

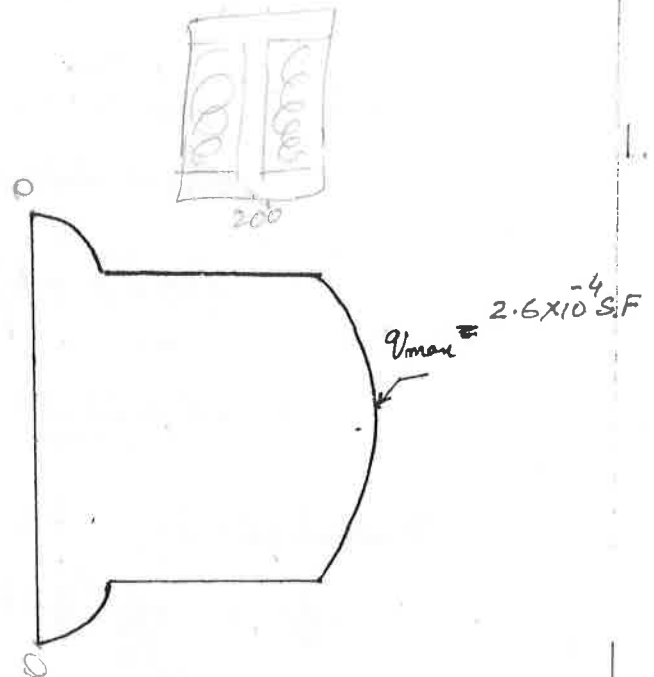
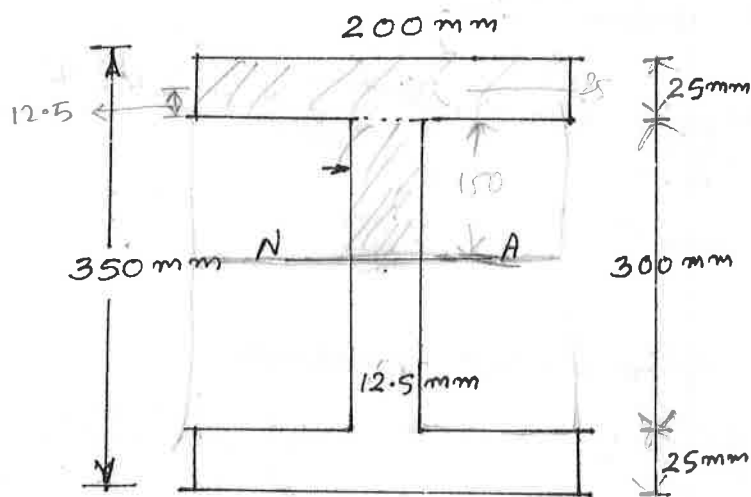


$$W \quad V_A =$$

after
Prob 3:
mm

Calculate the ratio of maximum to mean shear stress in an I-beam 200 mm wide and 350 mm deep, having the flanges 25 mm thick and web 12.5 mm thick. Also find the percentage of the total shearing force carried by the web.

Solution:



$$\text{Shear stress } q = \frac{SF}{Ib} (A\bar{y})$$

Shear stress at extreme top and bottom $q = 0$.

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{200 \times 350^3}{12} - \frac{187.5 \times 300^3}{12}$$

{ 187.5 = 200 - 12.5 }

$$I = 292.708 \times 10^4 \text{ mm}^4$$

→ Maximum Shear Stress:

Maximum Shear stress occur at Neutral Axis

$$q_{max} = \frac{SF}{Ib} (A\bar{y})$$

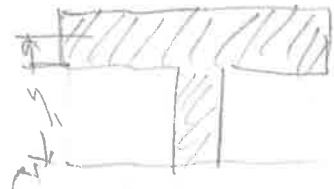
$$= \frac{SF}{292.708 \times 10^6 \times 12.5} (200 \times 25 + 150 \times 12.5) \bar{y}$$

where $\bar{y}$ = Distance of Centroid of AREA above NA.

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(200 \times 25 \times 162.5) + (150 \times 12.5 \times 75)}{200 \times 25 + 150 \times 12.5}$$

$$= \frac{953125}{6875}$$

$$\bar{y} = \underline{138.64 \text{ mm.}}$$



Substituting $\bar{y}$ in above equation

$$q_{max} = \frac{SF}{292.708 \times 10^6 \times 12.5} (200 \times 25 + 150 \times 12.5) \times 138.64$$

$$q_{max} = 2.6 \times 10^{-4} \times SF$$

→ Average Shear Stress $q_{av} = \frac{SF}{\text{Area of CS.}}$

$$q_{av} = \frac{SF}{(200 \times 25 \times 2 + 12.5 \times 300)}$$

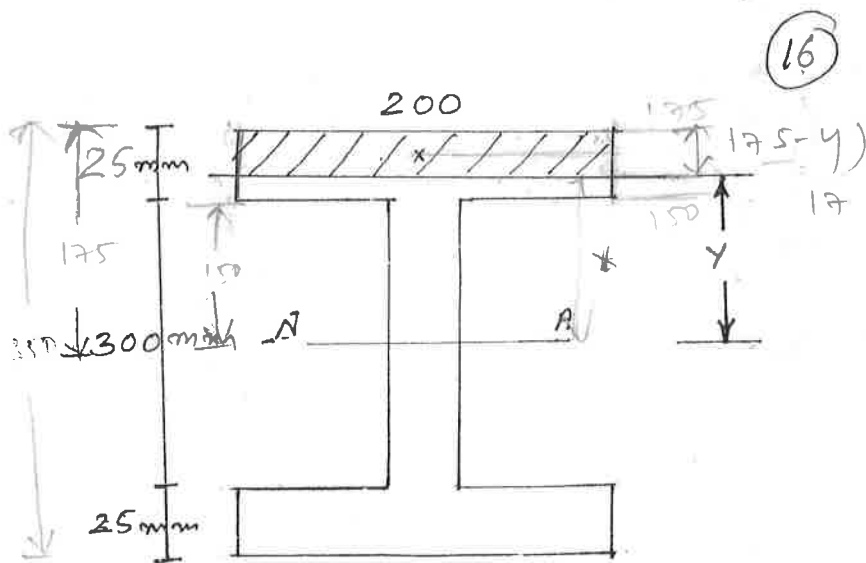
$$= \underline{7.27 \times 10^{-5} \times SF}$$

$$\rightarrow \text{Ratio } \frac{q_{max}}{q_{av}} = \frac{2.6 \times 10^{-4} \times SF}{7.27 \times 10^{-5} \times SF} = \underline{3.57}$$

Shear force carried by web SF_w .

$$SF_w = (\text{Total SF}) - (\text{Shear force taken by flanges})$$

$$= SF - SF_f$$



Shear force taken by flange:

Consider the section x-x at a distance 'y' from NA

→ Shear stress at x-x $q = \frac{SF}{I b} (A \bar{y})$.

where $I = 292.708 \times 10^4 \text{ mm}^4$

$b = 200 \text{ mm}$.

$A \bar{y} = 200 (175 - y) \left(y + \frac{175 - y}{2} \right)$.

$= 200 (175 - y) \left(\frac{2y + 175 - y}{2} \right)$.

$= 200 (175 - y) \frac{(175 + y)}{2}$

$A \bar{y} = 100 (175^2 - y^2)$.

Substituting I , b and $A \bar{y}$ in Eqn of 'q'

$q = \frac{SF}{292.7 \times 10^4 \times 200} 100 (175^2 - y^2)$

$= \frac{SF}{585.4 \times 10^4} (175^2 - y^2)$

→ Shear force in an elemental strip in flange of thickness dy and width 200 mm.

$SF_{\text{strip}} = q \times \text{area of strip}$

$= \frac{SF}{585.4 \times 10^4} (175^2 - y^2) \times 200 dy$.

Shear force in flange

$$SF_f = \int_{150}^{175} \frac{SF}{584.4 \times 10^6} (175^2 - y^2) 200 dy$$

$$\int 1 dy = y$$

$$y^n = \frac{y^{n+1}}{n+1}$$

$$\frac{y^{n+1}}{n+1}$$

$$\frac{y^2}{2}$$

$$= \frac{SF}{2922000} \int_{150}^{175} (175^2 - y^2) dy$$

$$= \frac{SF}{2922000} \left[175^2 y - \frac{y^3}{3} \right]_{150}^{175}$$

$$= \frac{SF}{292200} \left[\left(175^2 \times 175 - \frac{175^3}{3} \right) - \left(175^2 \times 150 - \frac{150^3}{3} \right) \right]$$

$$S.F_f = \frac{SF}{2922000} (3572916.67 - 3468750)$$

$$= 0.0356 SF$$

$$\left. \begin{array}{l} \text{Shear force taken by} \\ \text{two flanges} \end{array} \right\} = 2 \times 0.0356 \times SF = \underline{0.071 SF}$$

$$\left. \begin{array}{l} \text{Shear force taken} \\ \text{by web} \end{array} \right\} SF_w = 1SF - 0.071 SF = \underline{0.929 SF}$$

$$\left. \begin{array}{l} \text{Percentage of shear} \\ \text{force carried by web} \end{array} \right\} = \frac{SF_w}{SF} \times 100 = \frac{0.929 SF}{SF} \times 100 = \underline{92.9\%}$$

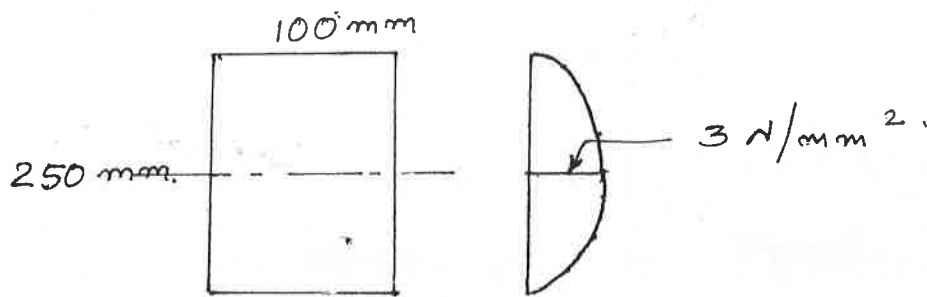
(18)

✓ Prob 4: JNTU May/June 2009. Suppl. Set no (4).

A rectangular beam 100 mm wide and 250 mm deep is subjected to a maximum shear force of 50 kN. Determine

- Average shear stress.
- Maximum shear stress.
- Shear stress at a distance of 25 mm above the neutral axis

Solution:

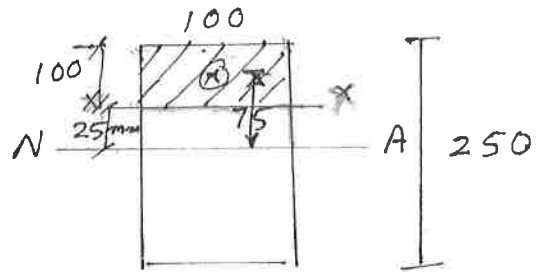


$$\begin{aligned}
 \text{a) Average Shear Stress } q_{av} &= \frac{SF}{bd} \\
 &= \frac{50 \times 10^3}{100 \times 250} \\
 &= \underline{\underline{2 \text{ N/mm}^2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) Max Shear Stress } q_{max} &= 1.5 \times q_{av} \quad (\text{For rect. Section}) \\
 &= 1.5 \times 2 \\
 &= \underline{\underline{3 \text{ N/mm}^2}}
 \end{aligned}$$

③ Shear stress at a distance 25 mm above the NA.

$$q = \frac{SF}{Ib} (A\bar{y})$$



$$I = \frac{bd^3}{12}$$

$$= \frac{100 \times 250^3}{12}$$

$$= \underline{130.20 \times 10^6 \text{ mm}^4}$$

$$b = \underline{100 \text{ mm}}$$

$A\bar{y}$ = Moment of portion above x-x about NA.

$$= (100 \times 100) \times 75$$

$$= \underline{75 \times 10^4 \text{ mm}^3}$$

$$\left(\frac{100}{2} + 25 \right)$$

Substituting I , b and $A\bar{y}$ in Eqn of q .

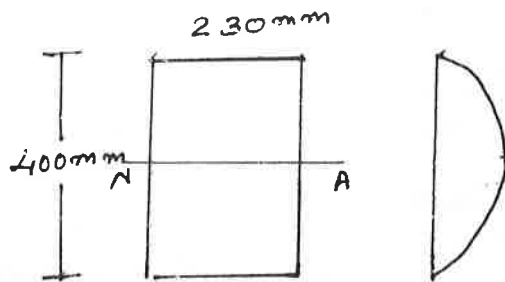
$$q = \frac{50 \times 10^3}{130.2 \times 10^6 \times 100} (75 \times 10^4)$$

$$= \underline{2.88 \text{ N/mm}^2}$$

✓ Prob 5: JNTU NOV 2007 Regular Set No (1).

Obtain the shear stress distribution for a rectangular cross section $230 \times 400 \text{ mm}$ subjected to a shear force of 40 kN . Calculate maximum and average shear stress.

Solution:



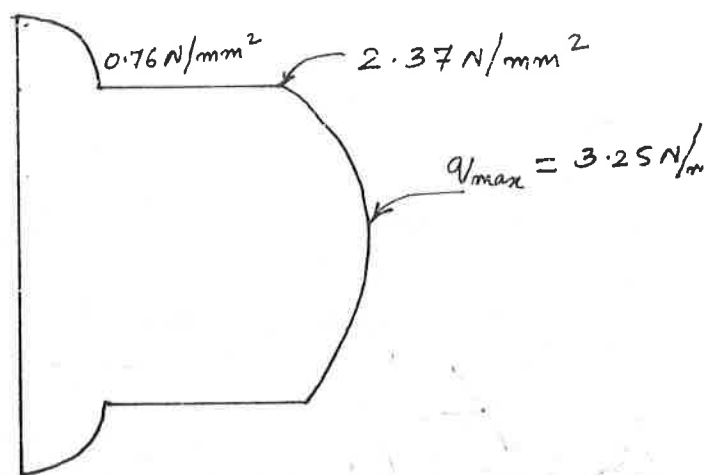
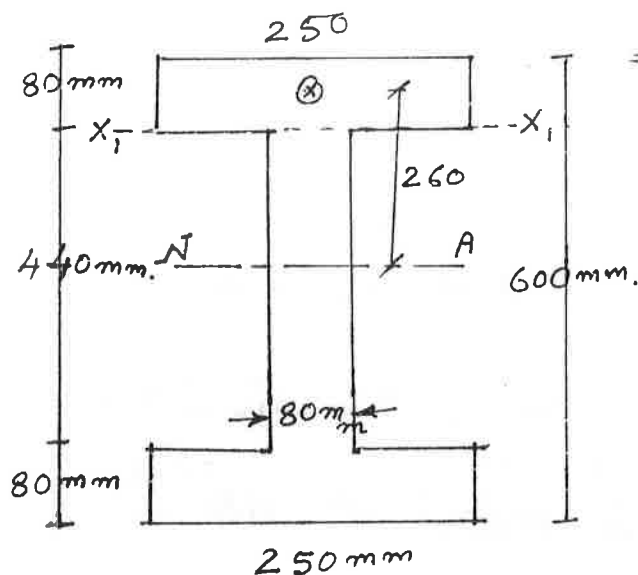
① Shear stress distribution for rectangular section } - Refer page (4).

$$\begin{aligned} \text{② Average Shear Stress } q_{av} &= \frac{SF}{bd} \\ &= \frac{40 \times 10^3}{230 \times 400} \\ &= \underline{0.435 \text{ N/mm}^2} \end{aligned}$$

$$\begin{aligned} \text{③ Maximum Shear Stress } q_{max} &= 1.5 q_{av} \\ &= 1.5 \times 0.435 \\ &= \underline{0.65 \text{ N/mm}^2} \end{aligned}$$

[lets see]
prob 6:

An I-section shown in figure is subjected to a
 $SF = 120 \text{ kN}$. sketch the shear stress distribution.
 Obtain maximum and mean shear stress.



SOLUTION:

Shear stress distribution $q = \frac{SF}{I b} (A \bar{y})$

→ Shear stress in flange } $q_{x_1-x_1} = \frac{120 \times 10^3}{I \times 250} (A \bar{y})$
 at the level $x_1 - x_1$

where $M I$ $I = \frac{B D^3}{12} - \frac{b d^3}{12} =$

$$= \frac{250 \times 600^3}{12} - \frac{170 \times 440^3}{12}$$

$$= 3.293 \times 10^9 \text{ mm}^4$$

$$A \bar{y} = (250 \times 80) \times 260$$

$$= 5.2 \times 10^6$$

(22)

Substituting I and $A\bar{Y}$ in Eqn of $q_{x_1-x_1}$,

$$q_{x_1-x_1} = \frac{120 \times 10^3}{3.293 \times 10^9 \times 250} \times 5.2 \times 10^6 \text{ (in flange)}$$

$$= \underline{0.76 \text{ N/mm}^2}$$

→ Shear Stress at x_1-x_1 in web.

$$q_{x_1-x_1} = \frac{120 \times 10^3}{3.293 \times 10^9 \times 80} \times 5.2 \times 10^6$$

(web $b = 80 \text{ mm}$)

$$= \underline{2.37 \text{ N/mm}^2}$$

→ Maximum Shear Stress occurs at NA.

$A\bar{Y}$ for portion above NA.

$$A\bar{Y} = a_1 y_1 + a_2 y_2 = (250 \times 80) 260 + (80 \times 220) \times 110$$

$$= 7.136 \times 10^6$$

$$\text{Shear Stress } q_{NA} = \frac{120 \times 10^3}{3.293 \times 10^9 \times 80} \times 7.136 \times 10^6$$

$$q_{NA} = \underline{3.25 \text{ N/mm}^2}$$

(max)

→ Average Shear Stress

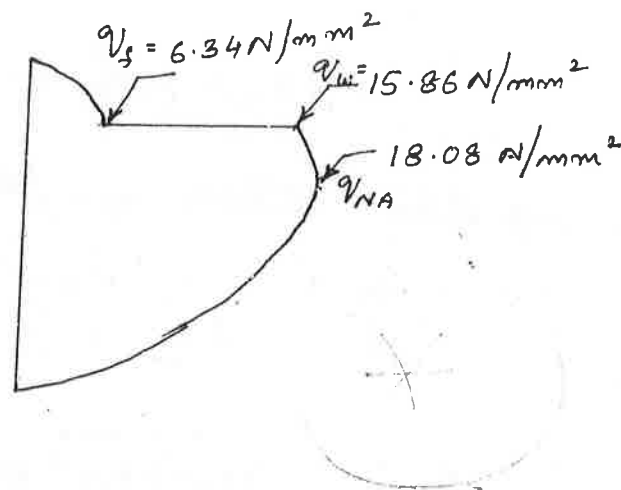
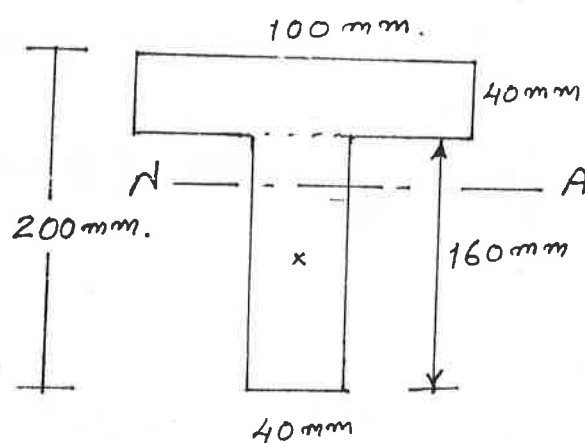
$$q_{AV} = \frac{SF}{\text{Area}} = \frac{120 \times 10^3}{(250 \times 80) + (440 \times 80) \times 2}$$

$$q_{AV} = \underline{2.17 \text{ N/mm}^2}$$

after

Prob 7: JNTU NOV 2007 Regular Set NO (3).

For a T-section with dimensions flange width 100 mm, Depth = 200 mm and uniform thickness of 40 mm. Obtain the shear stress distribution and calculate maximum and average shear stresses if it is subjected to a S.F = 100 kN.



Solution:

$$\text{Shear Stress } q = \frac{S.F}{I} (A \bar{y})$$

$$\text{Centroid from top } \bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

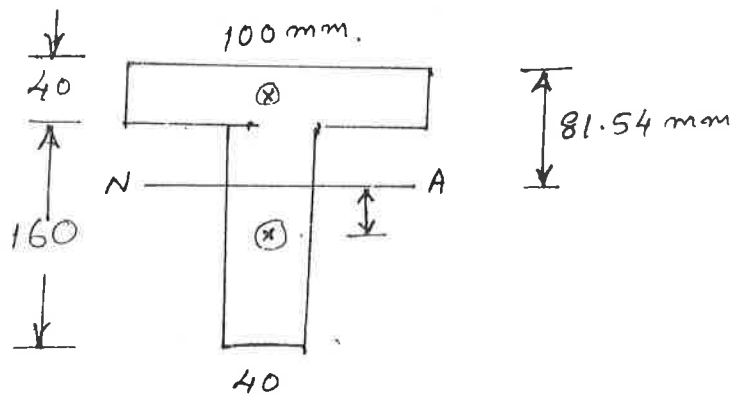
for Complete
T-Section

$$= \frac{(100 \times 40) \times 20 + (160 \times 40) \times 120}{100 \times 40 + 160 \times 40}$$

$$= \frac{848000}{10400}$$

$$= \underline{81.54 \text{ mm}}$$

(24)



MI "I" about NA. $I = \frac{b_1 d_1^3}{12} + a_1 d_1^2 + \frac{b_2 d_2^3}{12} + a_2 d_2^2$

$$I = \frac{100 \times 40^3}{12} + (100 \times 40) 61.54^2 + \frac{(40 \times 160^3)}{12} + (40 \times 160) (120 - 81.54)^2$$

$$= \underline{38.80 \times 10^6 \text{ mm}^4}$$

→ Shear stress in flange at the junction ($b = 100 \text{ mm}$)

$$q_f = \frac{S.F}{Ib} (A\bar{y})$$

$$= \frac{100 \times 10^3}{38.80 \times 10^6 \times 100} (100 \times 40 \times 61.54)$$

$$= \underline{6.34 \text{ N/mm}^2}$$

→ Shear stress in web at the junction ($b = 40 \text{ mm}$)

$$q_w = \frac{100 \times 10^3}{38.80 \times 10^6 \times 40} (100 \times 40 \times 61.54)$$

$$= \underline{15.86 \text{ N/mm}^2}$$

→ Shear stress at NA. ($b = 40 \text{ mm}$)

$$q_{NA} = \frac{100 \times 10^3}{38.8 \times 10^6 \times 40} (100 \times 40 \times 61.54 + 41.54 \times 40 \times \frac{41.54}{2})$$

$$q_{NA} = 18.08 \text{ N/mm}^2$$

→ Average sh. stress $q_{av} = \frac{100 \times 10^3}{(100 \times 40 + 160 \times 40)} = \underline{9.61 \text{ N/mm}^2}$

Align signs

① Shear stress $q = \frac{S.F}{I b} (\bar{A} \bar{y})$

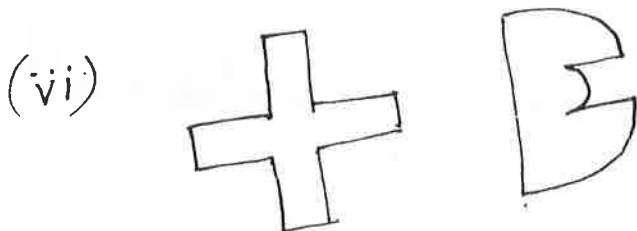
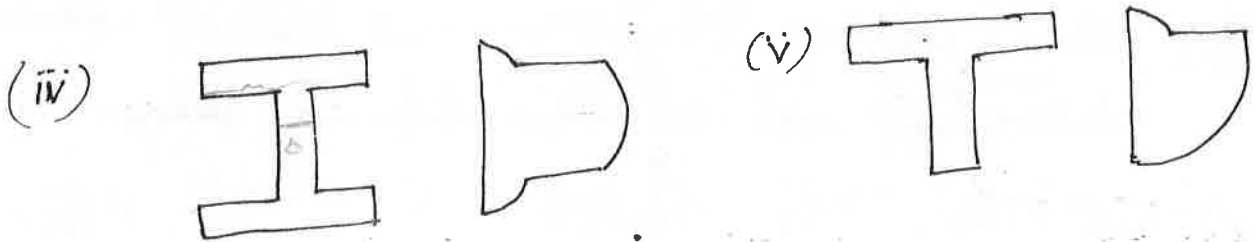
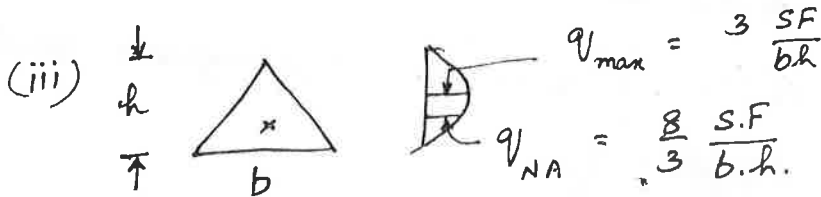
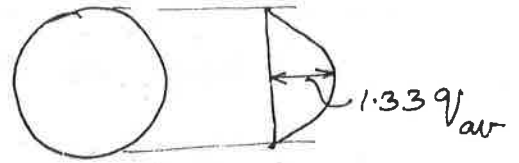
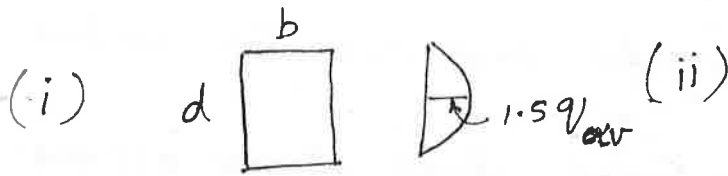
② Maximum Shear stress in rectangular section $q_{\max} = 1.5 q_{\text{ave}}$

③ Maximum shear stress in circular section $q_{\max} = 1.33 q_{\text{average}}$

④ Triangular section $q_{\max} = 3 \times \frac{S.F}{b.h}$ (occurs at $\frac{h}{2}$)

$q_{NA} = \frac{8}{3} \frac{S.F}{b.h}$ (occurs at $\frac{h}{3}$ from base)

⑤ Shear stress distribution pattern in various section.



$q \propto \frac{1}{b}$

Q 1: what is the ratio of max. to average shear stress in rectangular and circular sections?

Ans 1: In rectangular section $q_{max} = 1.5 q_{average}$
In circular section $q_{max} = 1.33 q_{average}$.

Q 2: where does maximum shear stress occur in rectangular and circular sections?

Ans 2: Maximum shear stress occurs at centroidal axis in rectangular and circular section.

Q 3: Where does maximum shear stress occurs in triangular section?

Ans 3: Maximum shear stress in triangular section occurs at height $\frac{h}{2}$.

Q 4: In I-section the approximate ratio of shear taken by web to shear taken by flanges is

(a) 50% : 50% (b) 60% : 40% (c) 90% : 10%.

Ans 4: (c) 90% : 10%.

Q 5: write equation for variation of shear stress 'q' across a section.

Ans 5: $q = \frac{S.F}{Ib} (A\bar{y})$ where S.F = Shear force.
I = MI of whole section about centroidal axis
b = width of section under consideration
 $A\bar{y}$ = Moment of area of portion above section considered.

TUTORIAL PROBLEMS.

① A beam of rectangular cross-section measures $100 \text{ mm} \times 200 \text{ mm}$. The beam is subjected to a shear force of 10 kN . Find (a) Average shear stress

(b) Maximum shear stress.

(c) Draw the variation of shear stress across the section.

(Ans (a) $q_{av} = 0.5 \text{ N/mm}^2$
(b) $q_{max} = 0.75 \text{ N/mm}^2$)

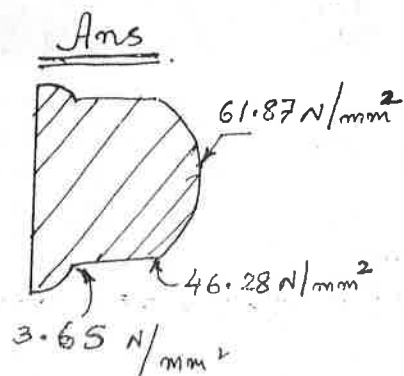
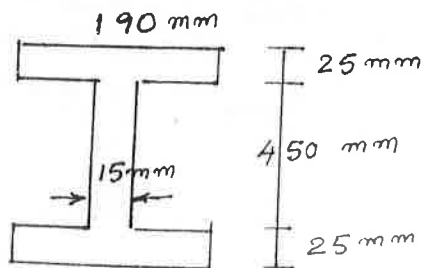
② A beam of circular section of dia 200 mm is subjected to a S.F of 12 kN . Find

(a) Average shear stress q_{av}

(b) Maximum shear stress q_{max} .

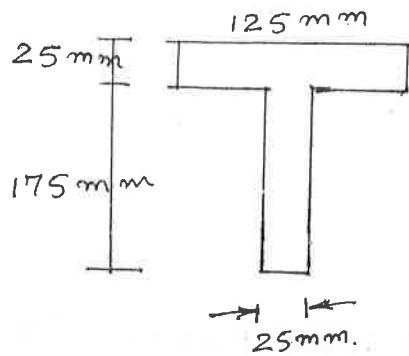
Ans : (a) $q_{av} = 0.38 \text{ N/mm}^2$
(b) $q_{max} = 0.51 \text{ N/mm}^2$

③ A beam of I-section shown in figure below is subjected to a S.F = 400 kN . Draw the variation of shear stress across the depth of section at Junction of flange at web and at Neutral axis.



$I_{xx} = 650 \times 10^6 \text{ mm}^4$

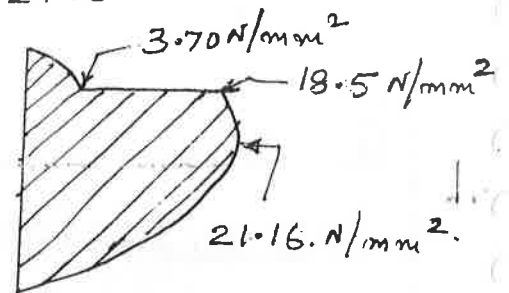
- ④ A beam of T-section is subjected to a S.F = 75 kN. Draw the shear stress across the section if the dimensions are as shown in figure.



Ans:

$$\bar{y} = 129.16$$

$$I_{xx} = 29.56 \times 10^6 \text{ mm}^4$$



- ⑤ A beam of triangular section of base $b = 150 \text{ mm}$ and height $h = 90 \text{ mm}$. The section is subjected to a S.F = 3 kN find

(a) ~~shear~~ ^{stress} at neutral axis q_{NA}

(b) ~~shear~~ ^{Max. shear stress} q_{max} .

Ans:

$$q_{NA} = 0.59 \text{ N/mm}^2$$

$$q_{max} = 0.67 \text{ N/mm}^2$$

Unit V DEFLECTION OF BEAMS.

①

5.1. Beam Bending into a circular arc :

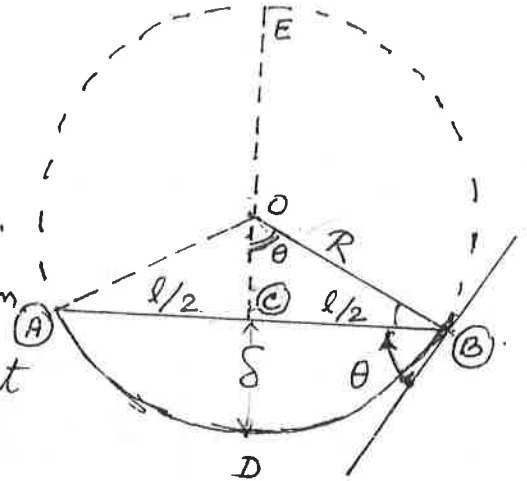
Consider a beam AB bending into circular arc as shown in figure 5.1 below.

Let l = span of beam AB.

R = Radius of bend arc.

M = Moment acting on beam

δ = Deflection in beam at centre. (= CD).



We know from co-ordinate geometry

$$DC \times CE = AC \times CB.$$

$$\delta \times (2R - \delta) = \frac{l}{2} \times \frac{l}{2}$$

$$2R\delta - \delta^2 = \frac{l^2}{4}$$

For practical beams 'Deflection' δ is negligibly small, and quantity δ^2 is still smaller, therefore it can be neglected.

$$\Rightarrow 2R\delta = \frac{l^2}{4} \quad \text{or}$$

$$\delta = \frac{l^2}{8R}$$

But from earlier chapter we know $\frac{M}{I} = \frac{E}{R}$ or $\boxed{\frac{1}{R} = \frac{M}{EI}}$

Substituting above $\delta = \frac{l^2}{8} \frac{M}{EI}$

$$\delta = \frac{Ml^2}{8I}$$

(2)

Let θ = Slope of deflected beam at A or B.

From fig $\hat{CBO} = 90 - \theta$. Sum of the angles in a triangle = 180°
 $\Rightarrow \hat{BOC} = \theta$. $90 + (90 - \theta) + \theta = 180$
 $\theta = \theta$

$$\Rightarrow \sin \theta = \frac{BC}{OB} = \frac{l/2}{R} = \frac{l}{2R}$$

opp/hyp

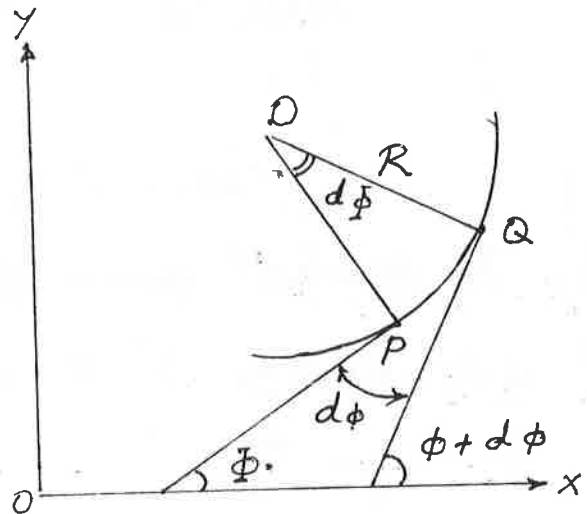
practically θ is very small $\Rightarrow \sin \theta = \theta$ in radians

$$\Rightarrow \theta = \frac{l}{2R} \quad \text{Substituting } \frac{1}{R} = \frac{M}{EI}$$

$\theta = \frac{Ml}{2EI}$

Eqn (1)

5.2 Slope, deflection and radius of curvature:



Consider an elemental length PQ of the arc AB of previous derivation.

Length of PQ = ds

Let ϕ and $\phi + d\phi$ be the angles subtended by tangents (at 'P' & 'Q') on x-axis.

(3)

→ Normals at 'P' and 'Q' meet at "O", which is the centre of curvature.

→ From fig $PO = QO = R$ (Radius of curvature)

We know Length of curve = Radius $\times$ angle subtended at centre

$$\Rightarrow ds = R \times d\phi$$

$$\text{or } \boxed{R = \frac{ds}{d\phi}} \text{ ————— Eqn (2)}$$

⇒ If co-ordinates of P are (x, y)

$$\text{then } \tan \phi = \frac{dy}{dx} \text{ — (a) opp/adj}$$

$$\sin \phi = \frac{dy}{ds} \text{ — (b) opp/hyp}$$

$$\cos \phi = \frac{dx}{ds} \text{ — (c) adj/hyp}$$

Radius of curvature 'R' may be written as $\frac{\text{hyp}}{\text{adj}}$

$$\text{From Eqn (2) } R = \frac{ds}{d\phi} = \frac{ds/dx}{d\phi/dx} = \frac{\sec \phi}{d\phi/dx} \text{ ————— (3)}$$

→ Differentiating Eqn (a) with respect to x.

$$\tan \phi = \frac{dy}{dx}$$

$$\text{Diff } \sec^2 \phi \frac{d\phi}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{d\phi}{dx} = \frac{d^2y/dx^2}{\sec^2 \phi} \text{ Substitute in (3)}$$

$$\begin{aligned}
 R &= \frac{\sec \phi}{d\phi/dx} \quad (4) \\
 &= \frac{\sec \phi}{\frac{d^2y/dx^2}{\sec^2 \phi}}
 \end{aligned}$$

$$R = \frac{\sec^3 \phi}{d^2y/dx^2}$$

$$\text{or } \frac{1}{R} = \frac{d^2y/dx^2}{\sec^3 \phi}$$

$$= \frac{d^2y/dx^2}{(\sec^2 \phi)^{3/2}}$$

$$= \frac{d^2y/dx^2}{(1 + \tan^2 \phi)^{3/2}}$$

For practical beam slope $\tan \phi$ is small, hence $\tan^2 \phi$ may be neglected.

$$\Rightarrow \frac{1}{R} = \frac{d^2y}{dx^2}$$

$$\text{But from Eqn } \frac{M}{I} = \frac{E}{R} \Rightarrow \frac{1}{R} = \frac{M}{EI}$$

$$\Rightarrow \frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\text{or } \boxed{M = EI \frac{d^2y}{dx^2}}$$

Double integration of

Double integration

⇒ The above equation is the "Differential Equation for Elastic line of a Beam".

⇒ Double integration of this equation obtains the deflection ' y ' of the beam. Hence the method is known as "Double Integration Method".

Sum

⇒ Single integration of the equation obtains the slope $\frac{dy}{dx}$.

5.3 Determination of Slope and Deflection for Cantilever beams subjected to :

(i) point load

(ii) u.d.l.

(iii) uniformly varying load.

(3)

(6)

Deflection of Cantilever beam carrying point load at the free end :

Figure shows a Cantilever beam AB

Let l = Span of beam.

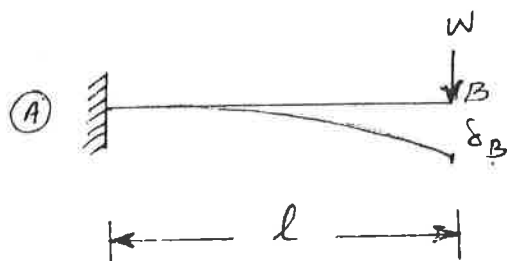
W = Load applied at free end B.

I = MI of beam about NA.

E = Modulus of elasticity of material of beam.

The beam is of uniform cross section.

Consider a section $x-x$ at a distance ' x ' from fixed end.



$$\Rightarrow \text{BM at } x-x \quad M_x = W(l-x) \quad \text{--- (1)}$$

$$\text{But from deflection equation } M = EI \frac{d^2y}{dx^2} \quad \text{--- (2)}$$

Equating (1) and (2).

$$EI \frac{d^2y}{dx^2} = W(l-x)$$

$$\text{Integrating } EI \frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right) + C_1 \quad \text{--- (3)}$$

where C_1 = Constant of integration.

$$\Rightarrow \text{At A slope } \frac{dy}{dx} = 0 \quad \text{and } x = 0.$$

Substituting in (3) $C_1 = 0$.

$$\Rightarrow EI \frac{dy}{dx} = W \left(lx - \frac{x^2}{2} \right) \text{ ————— (4)}$$

$$\text{Integrating again } EI y = W \left(\frac{lx^2}{2} - \frac{x^3}{6} \right) + C_2 \text{ — (5)}$$

where C_2 = Constant of integration.

Again at A deflection $y=0$ and $x=0$

Substituting in (5) $C_2 = 0$.

$$\Rightarrow EI y = W \left(\frac{lx^2}{2} - \frac{x^3}{6} \right) \text{ ————— (6)}$$

From Eqn (4) and Eqn (6) $\frac{dy}{dx} = \theta$ (Slope)
 $y = \delta$ (Deflection).

$\Rightarrow$ At B, $x=l$ Substituting in (4) Slope θ may be obtained

$$EI \theta_B = W \left(lx - \frac{x^2}{2} \right) \text{ — (4) } \left\{ \begin{array}{l} \text{Substituting} \\ \frac{dy}{dx} = \theta. \end{array} \right.$$

$$\theta_B = \frac{W}{EI} \left(lx - \frac{l^2}{2} \right)$$

$$\boxed{\theta_B = \frac{Wl^2}{2EI}} \text{ ————— (7)}$$

$\Rightarrow$ Substitute $x=l$ and $y=\delta_B$ in Eqn (6)

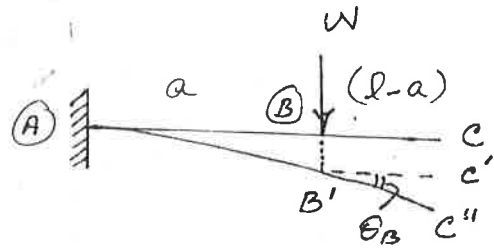
$$EI \delta_B = W \left(\frac{l \cdot l^2}{2} - \frac{l^3}{6} \right)$$

$$\boxed{\delta_B = \frac{Wl^3}{3EI}} \text{ ————— (8)}$$

Cantilever Beam Subjected to a Concentrated load 'W' at a distance 'a' from fixed end:

Figure shows Beam AC.

Load 'W' is applied at 'B'



Slope at 'B' $\theta_B = \frac{Wa^2}{2EI}$ $\rightarrow$ (From Eqn (7))

Deflection at 'B' $\delta_B = \frac{Wa^3}{3EI}$ (From Eqn (8))

$\rightarrow$ Bending of beam takes place between A and B whereas it remains straight between B and C since the BM is zero between B & C.

$\rightarrow$ Let B deflects to B' and C deflects to C''.

Deflection at C = $CC' + C'C''$

$\delta_C = \text{Deflection at B} + BC' \tan \theta_B$

$= \frac{Wa^3}{3EI} + (l-a) \theta_B$

($\tan \theta_B = \theta_B$ for small value of θ_B)

$$\delta_C = \frac{Wa^3}{3EI} + (l-a) \frac{Wa^2}{2EI}$$

(4)

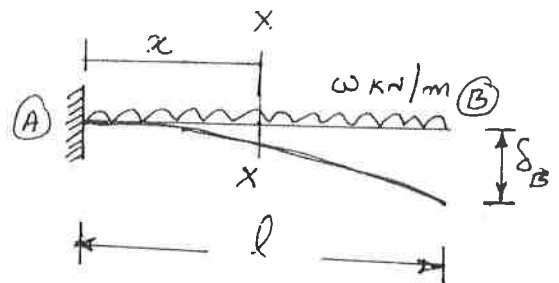
Cantilever Beam Subjected to uniformly distributed load w on whole length.

Figure shows Beam AB.

w kN/m load applied on whole span.

l = Span of beam.

I = MI of beam.



→ Consider a section x-x at a distance 'x' from (A).

$$\begin{aligned} \text{BM at } x-x \quad M &= -w(l-x) \frac{(l-x)}{2} \\ &= -w \frac{(l-x)^2}{2} \quad \text{--- (1)} \end{aligned}$$

$$\text{But we know } M = EI \frac{d^2 y}{dx^2} \quad \text{--- (2)}$$

$$\text{Equating (1) \& (2)} \quad EI \frac{d^2 y}{dx^2} = -w \frac{(l-x)^2}{2}$$

$$\text{Integrating} \quad EI \frac{dy}{dx} = +w \frac{(l-x)^3}{6} + C_1$$

→ At A ; $x = 0$ and slope $\frac{dy}{dx} = 0$.

$$\text{Substitute in above Eqn} \quad 0 = +w \frac{(l-0)^3}{6} + C_1$$

$$C_1 = -\frac{wl^3}{6}$$

$$\Rightarrow EI \frac{dy}{dx} = w \frac{(l-x)^3}{6} - \frac{wl^3}{6} \quad \text{--- (3) Slope Equation.}$$

Deflection of cantilever beam ^{carrying} with uniformly varying

load with zero intensity at fixed end and ' w ' at the free end.:

Beam shown in fig (a) may be represented by beam in fig (b) i.e udl acting

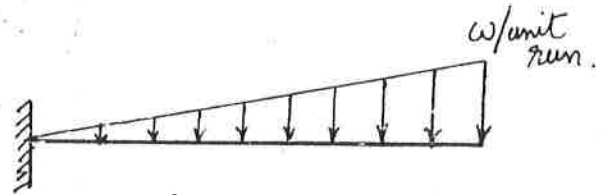
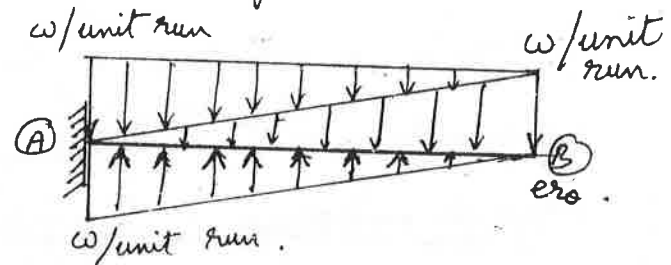


fig (a).



→ Deflection at B δ_B = Downward deflection due to udl
- upward deflection due to uvl.

$$= \frac{wl^4}{8EI} - \frac{wl^4}{30EI}$$

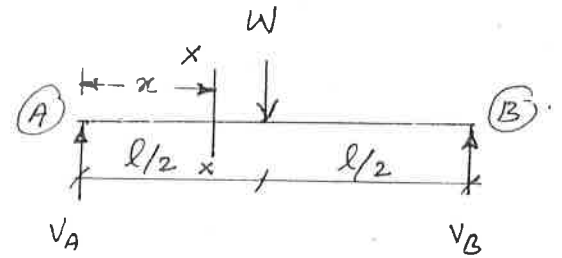
$$= \frac{15wl^4 - 4wl^4}{120EI}$$

$$\boxed{\delta_B = \frac{11}{120} \frac{wl^4}{EI}}$$

Deflection of Simply supported Beam carrying central point load

Consider the beam AB shown.

→ From symmetry $V_A = V_B = \frac{W}{2}$

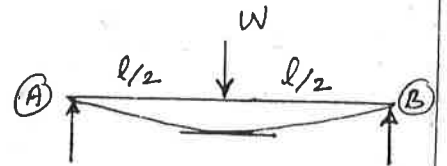


From double integration

$$\text{method } M = EI \frac{d^2 y}{dx^2} \quad \text{--- (1)}$$

Consider BM at x-x $M = \frac{W}{2} \cdot x$ --- (2)

Equate (1) & (2) $EI \frac{d^2 y}{dx^2} = \frac{W}{2} x$



Integrating $EI \frac{dy}{dx} = \frac{Wx^2}{4} + C_1$ --- (3)

→ At centre of span slope $\frac{dy}{dx} = 0$ and $x = \frac{l}{2}$

Substitute in above equation $0 = \frac{W}{4} \left(\frac{l}{2}\right)^2 + C_1$

$$C_1 = -\frac{Wl^2}{16}$$

Substituting in Eqn (3) $EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{Wl^2}{16}$ SLOPE EQUATION (4)

→ Integrating again $EI y = \frac{Wx^3}{12} - \frac{Wl^2 x}{16} + C_2$ --- (5)

At support 'A' $x = 0$ and deflection $y = 0$.

Substituting in Eqn (5) $0 = C_2$ i.e. $C_2 = 0$

(16)

Substituting $C_2 = 0$ in Eqn (5)

$$EI y = \frac{Wx^3}{12} - \frac{Wl^2x}{16} \quad \text{--- (6) DEFLECTION EQUATION}$$

→ Deflection at centre is maximum

At centre $x = \frac{l}{2}$, $y = \delta_c$ substitute in Eqn (6).

$$\begin{aligned} EI \delta_c &= \frac{W}{12} \left(\frac{l}{2}\right)^3 - \frac{Wl^2}{16} \cdot \frac{l}{2} \\ &= \frac{Wl^3}{96} - \frac{Wl^3}{32} \end{aligned}$$

$$\boxed{\delta_c = \frac{Wl^3}{48EI}}$$

Slope at mid span $\frac{dy}{dx} = \theta_c = 0$.

→ Slope at support θ_A , At 'A' $x = 0$.

Substituting in Eqn (4)

$$EI \theta_A = \frac{W}{4}(0) - \frac{Wl^2}{16}$$

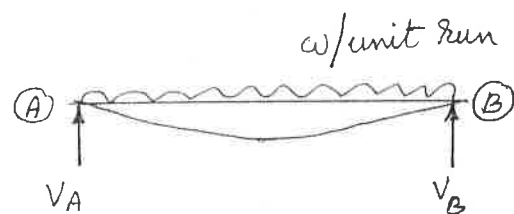
$$\boxed{\theta_A = -\frac{Wl^2}{16EI}}$$

(7)

Deflection of Simply Supported beam carrying uniformly distributed load 'w' on whole span:

Consider the beam AB.

→ Applying double integration method



$$M = EI \frac{d^2 y}{dx^2} \quad \text{--- (1)}$$

BM at distance 'x' from 'A' $M = V_A \cdot x - \frac{wx^2}{2}$

$$= \frac{wl}{2} \cdot x - \frac{wx^2}{2} \quad \text{--- (2)}$$

Equating (1) & (2) $EI \frac{d^2 y}{dx^2} = \frac{wl}{2} \cdot x - \frac{wx^2}{2}$

Integrating $EI \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1 \quad \text{--- (3)}$

At mid span $x = \frac{l}{2}$, and $\frac{dy}{dx} = 0$ Substituting in (3).

$$EI \theta_c = \frac{wl}{4} \left(\frac{l}{2}\right)^2 - \frac{w}{6} \left(\frac{l}{2}\right)^3 + C_1$$

$$0 = \frac{wl^3}{16} - \frac{wl^3}{48} + C_1$$

$$C_1 = -\frac{wl^3}{24} \quad \text{Substituting in (3)}$$

$$\rightarrow EI \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24} \quad \text{--- (4) SLOPE EQUATION.}$$

Integrating again $EI y = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl \cdot x}{24} + C_2$

At support $x=0$ and deflection $y=0$

$$\Rightarrow EI y = \frac{\omega x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega l^3 x}{24} + C_2$$

$$0 = C_2 \quad \text{ie } \boxed{C_2 = 0}$$

$$\Rightarrow EI y = \frac{\omega x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega l^3 x}{24} \quad \text{--- (5) DEFLECTION EQUATION.}$$

Deflection at centre $x = \frac{l}{2}$ substituting in (5)

$$EI y = \frac{\omega l}{12} \left(\frac{l}{2}\right)^3 - \frac{\omega}{24} \left(\frac{l}{2}\right)^4 - \frac{\omega l^3}{24} \left(\frac{l}{2}\right)$$

$$= \frac{\omega l^4}{96} - \frac{\omega l^4}{384} - \frac{\omega l^4}{48}$$

$$= -\frac{5}{384} \omega l^4$$

$$y = -\frac{5}{384} \frac{\omega l^4}{EI}$$

(-ve sign for downward deflection
and $y = \delta_c$)

$$\boxed{\delta_c = \frac{5}{384} \frac{\omega l^4}{EI}}$$

→ Slope at centre $\frac{dy}{dx} = \theta_c = 0$

→ Slope at Support A = Slope at Support B.

At 'A' $x=0$; Substituting in Eqn (4)

$$EI \frac{dy}{dx} = -\frac{\omega l^3}{24}$$

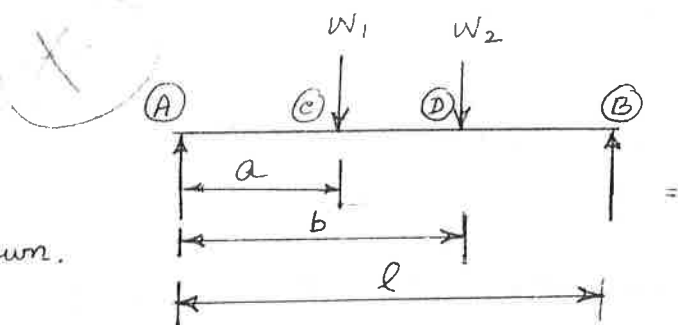
$$\left(\frac{dy}{dx} = \theta \right)$$

$$\boxed{\theta = -\frac{\omega l^3}{24EI}}$$

② MACAULAY'S METHOD

→ This method is convenient for finding deflection in beams subjected to point loads or discontinuous loads.

→ Consider the beam AB shown in figure subjected to loads w_1 and w_2 as shown.
Let l = span of beam.



Let V_A and V_B be the support reactions.

→ At any section at distance 'x' between A and C
Bending Moment: $M_x = V_A x$. ——— ①

The above moment holds good for all values between A and C.

→ At any section distance 'x' from 'A' and between C and D

Bending Moment: $M_x = V_A x - w_1(x-a)$ ——— ②

Equation ② of BM is applicable for $x=a$ to $x=b$.

→ At any section distance 'x' from 'A' and between D and B

Bending Moment: $M_x = V_A x - w_1(x-a) - w_2(x-b)$ ——— ③

Equation ③ of BM is applicable for $x=b$ to $x=l$.

→ The general equation for BM may be written as

$$M_x = EI \frac{d^2 y}{dx^2} = V_A x - w_1(x-a) - w_2(x-b) \text{ ——— ④}$$

In the above equation as the magnitude of 'x' increases the law of loading changes and additional expressions appears.

- For $x = 0$ to $x = a$ the first term of Eqn (4) is considered.
- For $x = a$ to $x = b$ first two terms of Eqn (4) are considered.
- For $x = b$ to $x = l$ all the terms of Eqn (4) are considered.

For integrating Eqn (4) to obtain slope and deflection

the following points shall be remembered

- (i) The constants of integration C_1 and C_2 should be written after the first term of Eqn (4).
- (ii) The quantities $(x-a)$ and $(x-b)$ should be integrated as $\frac{(x-a)^2}{2}$ and $\frac{(x-b)^2}{2}$ and NOT as $\frac{x^2}{2} - ax$ or $\frac{x^2}{2} - bx$.
- (iii) Constants C_1 and C_2 are valid for all values of 'x'.

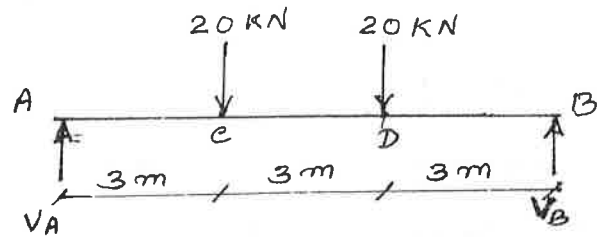
The constants C_1 and C_2 may be obtained by applying end conditions i.e. at $x = 0$ deflection $y = 0$ etc.

(21)

prob 1: JNTU May/June 2009 Supplementary Set No 1.

Determine the maximum deflection and the slope of the beam shown in figure using Macaulay's Method.

SOLUTION:



We know $M_x = EI \frac{d^2 y}{dx^2} = V_A \times x \quad \left| -20(x-3) \right| -20(x-6)$

To find V_A and V_B ; Taking moment about 'B'.

$$V_A \times 9 - 20 \times 6 - 20 \times 3 = 0 \Rightarrow V_A = 20 \text{ kN.}$$

Further $V_A + V_B = 40 \Rightarrow V_B = 20 \text{ kN.}$

Substituting V_A and V_B in standard equation

$$EI \frac{d^2 y}{dx^2} = 20 \times x \quad \left| -20(x-3) \right| -20(x-6)$$

Integrating $EI \frac{dy}{dx} = 20 \times \frac{x^2}{2} + C_1 \quad \left| -20 \frac{(x-3)^2}{2} \right| -20 \frac{(x-6)^2}{2}$

$$= 10x^2 + C_1 \quad \left| -10(x-3)^2 \right| -10(x-6)^2$$

Integrating again $EI y = 10 \frac{x^3}{3} + C_1 x + C_2 \quad \left| -10 \frac{(x-3)^3}{3} \right| -10 \frac{(x-6)^3}{3}$

(2)

At $x=0$; deflection $y=0$ Substituting above and considering first term only.

$$0 = C_2 \Rightarrow \boxed{C_2 = 0}$$

$$\text{Eqn (2) becomes } EI y = \frac{10x^3}{3} + C_1 x - 10 \frac{(x-3)^3}{3} - 10 \frac{(x-6)^3}{3}$$

Again at $x=9\text{m}$. deflection $y=0$; Substituting above.

$$\begin{aligned} 0 &= 10 \times \frac{9^3}{3} + C_1 \times 9 - 10 \times \frac{6^3}{3} - 10 \times \frac{3^3}{3} \\ &= 2430 + 9C_1 - 720 - 90 \end{aligned}$$

$$\boxed{C_1 = -180}$$

→ To find slope at support, substitute C_1 in Eqn (1)

$$EI \frac{dy}{dx} = 10x^2 - 180 - 10(x-3)^2 - 10(x-6)^2$$

$$\text{At Support } x=0 \Rightarrow EI \frac{dy}{dx} = -180 \quad (\text{consider only first term since } x < 3\text{m})$$

$$\text{Slope } \frac{dy}{dx} = \frac{-180}{EI}$$

→ Maximum Deflection: For max. defln $\frac{dy}{dx} = 0$ between C and D i.e. $x < 6\text{m}$.

$$\begin{aligned} \text{Substituting in (1) above } 0 &= 10x^2 - 180 - 10(x-3)^2 \\ &= 10x^2 - 180 - 10(x^2 + 9 - 6x) \\ &= 10x^2 - 180 - 10x^2 - 90 + 60x \end{aligned}$$

$$\boxed{x = 4.5\text{m.}} \quad \text{Substitute in Eqn (2)}$$

$$\begin{aligned} EI y_{\text{max.}} &= \frac{10x^3}{3} - 180x - 10 \frac{(x-3)^3}{3} \\ &= 10 \times \frac{4.5^3}{3} - 180 \times 4.5 - 10 \frac{(4.5-3)^3}{3} \end{aligned}$$

$$\text{Defln } y = 247.5/\text{EI}$$

Prob 2: JNTU May/June 2009 Suppl. set No (2)

A cantilever of 4m span length carries a load 40kN at its free end. If the deflection at the free end is not to exceed 8mm, what must be the moment of Inertia of cantilever section.

SOLUTION:

Given $l = 4\text{m} = 4 \times 10^3 \text{ mm}$.

$W = 40 \text{ kN} = 40 \times 10^3 \text{ N}$.

Deflection $y = \delta = 8 \text{ mm}$.

we know for a cantilever subjected to point load at free end deflection

$$\delta = \frac{wl^3}{3EI}$$

Substituting l , w and δ in above equation

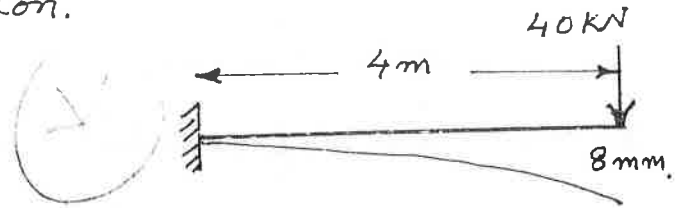
$$8 = \frac{40 \times 10^3 \times (4 \times 10^3)^3}{3E \times I}$$

$$I = \frac{1.067 \times 10^{14}}{E} \text{ mm}^4$$

If E is taken as $2 \times 10^5 \text{ M.pa}$.

$$I = \frac{1.067 \times 10^{14}}{2 \times 10^5}$$

$$I = 533.3 \times 10^6 \text{ mm}^4$$

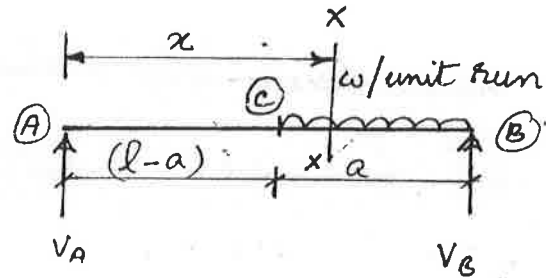


prob 3: JNTU May/June 2009 Suppl. Set no (2)

A horizontal beam of uniform section and length 'l' rests on supports at its ends. It carries a u.d.l w per unit length which extends over a length 'a' from the right hand support. Determine the value of 'a' in order that the maximum deflection may occur at the left hand end of the load, and if the maximum deflection is $\frac{wl^4}{KEI}$ determine the value of K.

SOLUTION:

Let V_A and V_B be support reactions



Applying Macaulay's method, consider section x-x.

$$EI \frac{d^2 y}{dx^2} = V_A x - \frac{w[x - (l-a)]^2}{2} \quad \text{--- (1)}$$

To find support reaction V_A , consider moment about 'B'.

$$\sum M_B = 0 \quad V_A l - \frac{w \cdot a^2}{2} = 0 \Rightarrow \boxed{V_A = \frac{wa^2}{2l}} \checkmark$$

→ Substituting V_A in eqn (1).

$$EI \frac{d^2 y}{dx^2} = \frac{wa^2}{2l} x - \frac{w}{2} [x - (l-a)]^2$$

$$\text{Integrating } EI \frac{dy}{dx} = \frac{wa^2 x^2}{4l} + C_1 - \frac{w}{6} [x - (l-a)]^3$$

Slope Equation (2)

Integrating again $EI y = \frac{wa^2 x^3}{12l} + C_1 x + C_2 \left| - \frac{w}{24} [x - (l-a)]^4 \right.$
 Deflection Equation (3)

At A, $x = 0$ and deflection $y = 0$; Substituting in Eqn (3)

$$\boxed{C_2 = 0}$$

At B; $x = l$ and deflection $y = 0$ substituting in Eqn (3)

$$0 = \frac{wa^2 l^3}{12l} + C_1 l - \frac{wa^4}{24}$$

$$C_1 = -\frac{wa^2 l}{12} + \frac{wa^4}{24l} \Rightarrow \boxed{C_1 = -\frac{wa^2}{24l} (2l^2 - a^2)}$$

→ Given maximum deflection occur at C. For maximum deflection $\frac{dy}{dx} = 0$ and at C, $x = (l-a)$

Substituting in (2) $EI \frac{dy}{dx} = \frac{wa^2 x^2}{4l} - \frac{wa^2}{24l} (2l^2 - a^2)$

$$\Rightarrow 0 = \frac{wa^2 (l-a)^2}{4l} - \frac{wa^2}{24l} (2l^2 - a^2)$$

$$\text{or } \frac{wa^2}{24l} [6(l-a)^2 - (2l^2 - a^2)] = 0$$

$$6(l^2 + a^2 - 2al) - 2l^2 + a^2 = 0$$

$$7a^2 - 12al + 4l^2 = 0$$

$$\Rightarrow a = \frac{12l \pm \sqrt{(12l)^2 - 4 \times 7 \times 4l^2}}{2 \times 7}$$

Solung $a = 0.45l$

Substituting in value of c_1 ~~xxx~~ $\omega(0.45l)^2$

$$c_1 = - \frac{\omega(0.45l)^2}{24l} [2l^2 - (0.45l)^2]$$

$$= - 0.0084\omega l (1.79l^2)$$

$$c_1 = \underline{-0.015\omega l^3}$$

Substituting c_1 and 'a' in deflection Eqn (2)

$$EI y_{\text{max}} = \frac{\omega a^2 x^3}{12l} - 0.015\omega l^3 x \quad (\text{where } x = l - a)$$

$$= \frac{\omega (0.45l)^2 (l - 0.45l)^3}{12l} - 0.015\omega l^3 (l - 0.45l)$$

$$= 0.034\omega l^4 - 0.0082\omega l^4$$

$$= - 0.0256\omega l^4$$

$$y_{\text{max}} = - \frac{0.0256\omega l^4}{EI}$$

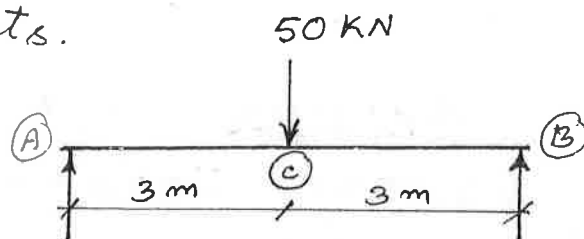
prob 4: JNTU May/June 2009 Suppl. Set No (3).

A beam 6 m long, simply supported at its ends, is carrying a point load of 50 kN at its centre. The MI of the beam is $78 \times 10^6 \text{ mm}^4$ If $E = 2.1 \times 10^5 \text{ N/mm}^2$

Calculate

(a) Deflection at centre of beam.

(b) Slope at the supports.



SOLUTION:

$$W = 50 \text{ kN} = 50 \times 10^3 \text{ N}$$

$$l = 6 \text{ m} = 6 \times 10^3 \text{ mm}$$

$$I = 78 \times 10^6 \text{ mm}^4$$

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

→ (a) Deflection at Centre $y = \delta = \frac{Wl^3}{48EI}$

$$y = \frac{50 \times 10^3 \times (6 \times 10^3)^3}{48 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$y = 13.74 \text{ mm}$$

(b) Slope at Supports

$$\text{Slope } \frac{dy}{dx} = \theta = -\frac{Wl^2}{16EI}$$

Substituting the values

$$= -\frac{50 \times 10^3 \times (6 \times 10^3)^2}{16 \times 2.1 \times 10^5 \times 78 \times 10^6}$$

$$= -6.86 \times 10^{-3} \text{ radians} \left(= -6.86 \times 10^{-3} \times \frac{180}{\pi} = -0.39^\circ \right)$$

Prob 5: JNTU May/June 2009 Suppl. Set No (4).

A simply supported beam of circular cross-section is 5m long and is of 150 mm dia. what will be the maximum value of the central load if the deflection of the beam does not 12.45 mm. Also calculate the slope at the supports. Take $E = 2 \times 10^8 \text{ KN/m}^2$.

SOLUTION:

$$l = 5 \text{ m} = 5 \times 10^3 \text{ mm}$$

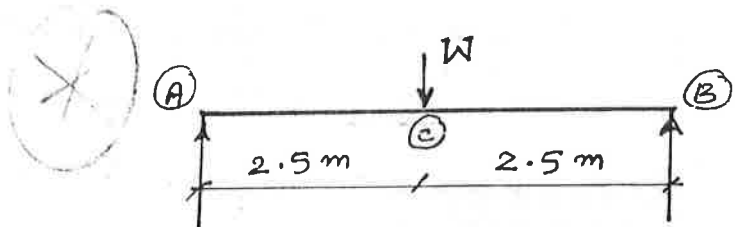
$$d = 150 \text{ mm.}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 150^4}{64} = 24.84 \times 10^6 \text{ mm}^4$$

$$y = \delta = 12.45 \text{ mm} \quad ; \quad E = \frac{2 \times 10^8 \times 10^3}{10^3 \times 10^3} \text{ N/mm}^2$$

$$W = ?$$

$$= 2 \times 10^5 \text{ N/mm}^2$$



→ Simply supported beam with central point load

$$\text{Defln. } y = \delta = \frac{Wl^3}{48EI}$$

$$12.45 = \frac{W(5 \times 10^3)^3}{48 \times 2 \times 10^5 \times 24.84 \times 10^6}$$

$$\boxed{W = 23.75 \times 10^3 \text{ N}} = 23.75 \text{ KN} \checkmark$$

→ Slope at supports

$$\text{slope } \frac{dy}{dx} = \theta = -\frac{Wl^2}{16EI} = \frac{23.75 \times 10^3 \times (5 \times 10^3)^2}{16 \times 2 \times 10^5 \times 24.84 \times 10^6}$$

$$= -7.46 \times 10^{-3} \text{ Radians}$$

$$= -7.46 \times 10^{-3} \times \frac{180}{\pi} = -0.43 \text{ degrees} \checkmark$$

Prob 6: JNTU May/June 2009 Suppl. NR

A 6.5 m long cantilever carries a uniformly distributed load over the entire length. If the slope at the free end is 1° (one degree), what is the deflection at free end.

SOLUTION:

$$l = 6.5 \text{ m} = 6.5 \times 10^3 \text{ mm.}$$

$$\frac{dy}{dx} = \theta = 1^\circ = 1 \times \frac{\pi}{180} \text{ radians}$$

$$= \underline{0.0174} \text{ radians.}$$

Let $w = \text{udl acting on the beam.}$

$$\text{Slope } \frac{dy}{dx} = \theta = -\frac{wl^3}{6EI}$$

$$0.0174 = -\frac{w \times (6.5 \times 10^3)^3}{6 \times EI}$$

$$w = -3.80 \times 10^{-13} EI$$

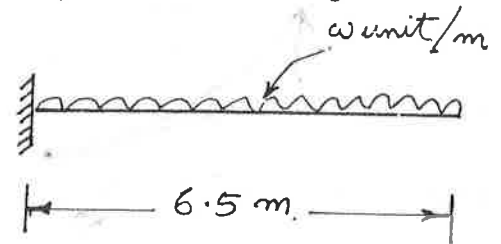
$$w = 3.80 \times 10^{-13} EI \text{ N/mm.}$$

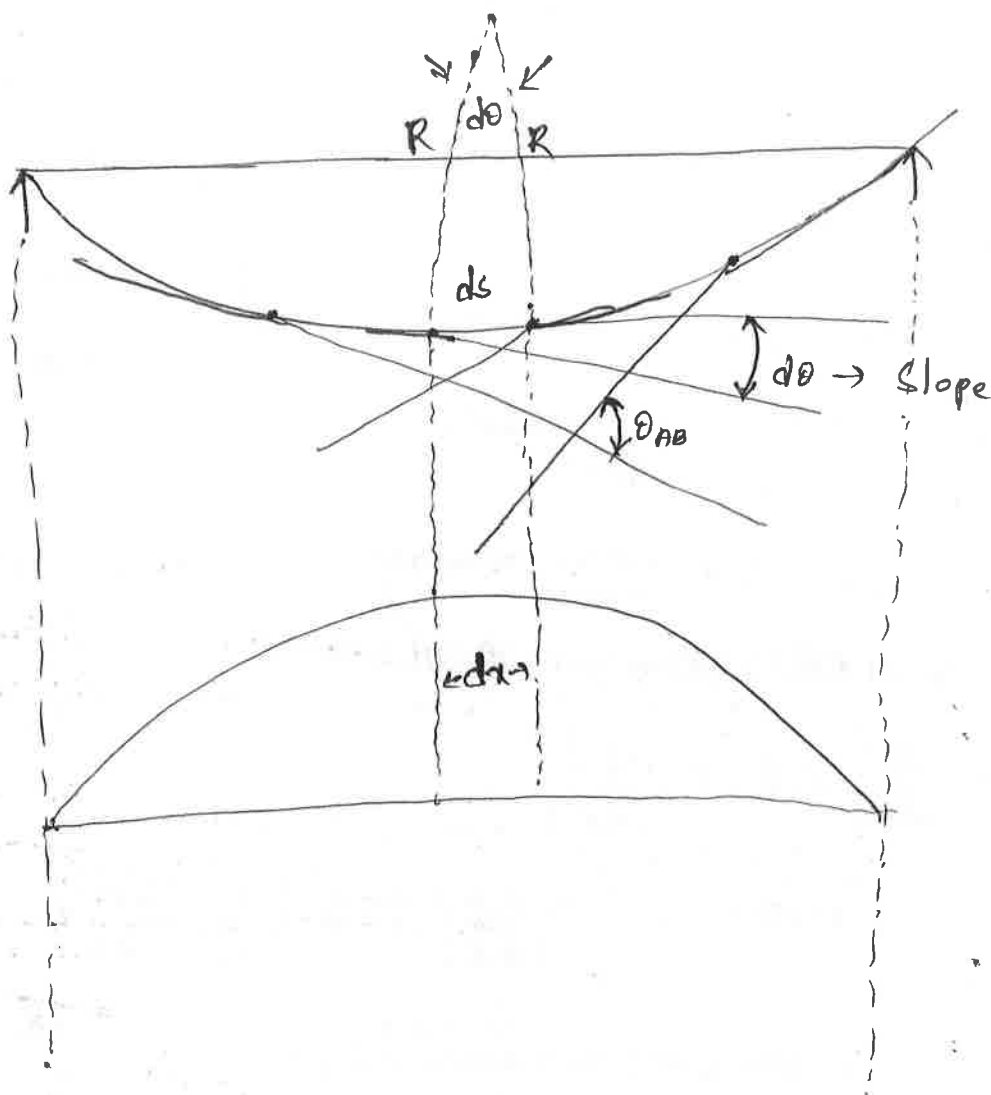
→ Deflection at free end.

$$y = \delta = \frac{wl^4}{8EI}$$

$$= \frac{3.80 \times 10^{-13} EI \times (6.5 \times 10^3)^4}{8EI}$$

$$\boxed{y = 84.79 \text{ mm.}}$$





Conjugate = A substance formed by reversible combination
 of two or more other
 = Mathematical value having reciprocal relation
 with another.

MOMENT AREA METHOD - MOHR'S THEOREMS (TO FIND DEFLECTION)

THEOREM 1 :

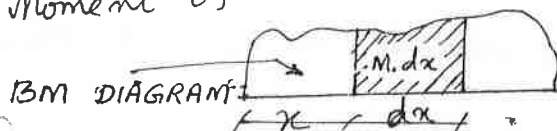
It states that slope, ^{at any point} in a beam is equal to $\frac{1}{EI} \times$ Area of BM diagram between point of zero slope and point under consideration.

$$\Rightarrow \text{Slope } \theta = \frac{1}{EI} \times \text{Area of BM diagram.}$$

THEOREM 2 :

It states that deflection at any point in a beam is equal to $\frac{1}{EI} \times$ moment of area of BM diagram between point of zero slope and point under consideration.

$$\Rightarrow \text{Deflection } y = \delta = \frac{1}{EI} \times \text{Moment of area of BM diagram.}$$



PROOF :

Consider sketch shown

Let A be the point of zero slope and zero deflection

A B is the deflected form of beam.

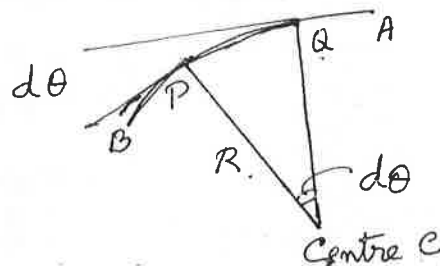
Let P and Q be two points on deflected curve,

x and $(x+dx)$ are distance of P and Q from point B.

Let $d\theta$ = Angle between tangents at P and Q.

= Angle between normals at P and Q.

R = Radius of curvature of arc PQ.



From figure $d\theta = \frac{PQ}{R}$ where $PQ = dx$.

$$= \frac{dx}{R}$$

Further from ^{Bending} Eqn $\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \Rightarrow \frac{1}{R} = \frac{M}{EI}$

Substituting above $d\theta = \frac{M}{EI} \cdot dx$ ————— (1)

Total Slope at B $\theta = \frac{1}{EI} \int_{x=0}^{x=BA} M \cdot dx$

$\int M \cdot dx$ = Area of BM diagram between A and B.

$$\therefore \text{Slope } \theta = \frac{1}{EI} \times \text{Area of BM diagram between A and B.}$$

→ Deflection due to bending of PQ

$dy = x \cdot d\theta$ substituting $d\theta$ from (1).

$$= x \cdot \frac{M}{EI} dx$$

$$= \frac{(M dx) x}{EI}$$

Total deflection at 'B' due to bending of AB.

$$\int dy = \int \frac{(M \cdot dx) x}{EI}$$

$$y = \frac{1}{EI} \int (M \cdot dx) \cdot x$$

Deflection

$$y = \frac{1}{EI} \times \text{Moment of BM diagram between A and B about 'B'}$$

Prob 7: JNTU Nov 2007 Regular. Set No (4).

(a) Explain the Mohr's theorems, for finding the slope and deflection of a beam.

(b) A simply supported 6m rolled steel joist carries a u.d.l of 10 kN/m length. Determine slope and deflection at a distance of 3m from one end of the beam.

Soln:

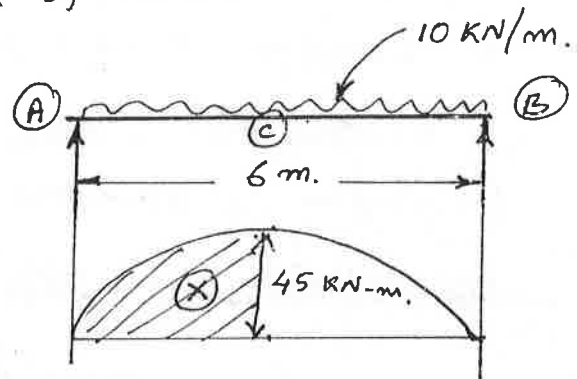
(a) Refer notes on derivation of Mohr's Theorem.

(b) $l = 6 \text{ m.}$

$w = 10 \text{ kN/m.}$

$$\text{BM } M = \frac{wl^2}{8} = \frac{10 \times 6^2}{8}$$

$$M = 45 \text{ kN-m.}$$



$$\bar{x} = \frac{5}{8} \times \frac{l}{2} = \frac{5}{8} \times 3 \rightarrow \text{Centroidal distance}$$

→ Slope at A $\theta_A = \frac{1}{EI}$ Area of BMD.

$$\begin{aligned} \text{Area } A &= \frac{2}{3} \times \frac{l}{2} \times \frac{wl^2}{8} = \frac{2}{3} \times 3 \times 45 \\ &= 90 \text{ m}^2. \end{aligned}$$

$$\theta_A = \frac{1}{EI} \times 90 = \frac{90}{EI} \text{ radians}$$

→ Deflection at Centre $y = \frac{1}{EI} \times \text{Moment of BM diag about 'A'}$

$$= \frac{1}{EI} \times 90 \times \left(\frac{5}{8} \times 3 \right)$$

$$y = \frac{168.75}{EI}$$

prob 8: JNTU Feb 2007 Suppl. set NR

Determine the deflection under the load and maximum deflection of the beam shown in figure below. Given $E = 210 \text{ GPa}$ and $I = 1000 \text{ cm}^4$.

SOLUTION:

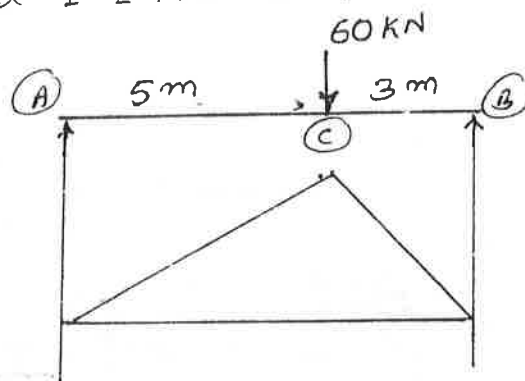
$$W = 60 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$a = 5 \text{ m}; \quad b = 3 \text{ m}.$$

$$E = 210 \text{ GPa} = 2.10 \times 10^5 \text{ N/mm}^2$$

$$I = 1000 \text{ cm}^4 = 1000 \times 10^4 \text{ mm}^4.$$

$$l = 8 \text{ m} = 8000 \text{ mm}.$$



→ Deflection under the load is given by the equation

$$y = \delta_c = \frac{W a^2 b^2}{3 E I l}$$

$$= \frac{60 \times 10^3 \times 5000^2 \times 3000}{3 \times 2.10 \times 10^5 \times 1000 \times 10^4 \times 8000}$$

$$\delta_c = \underline{267.85 \text{ mm}}$$

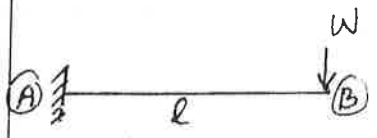
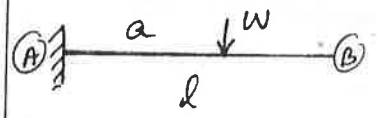
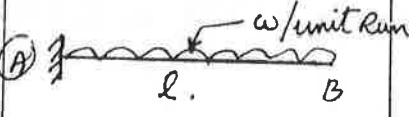
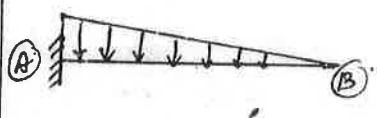
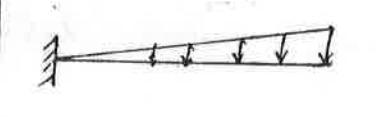
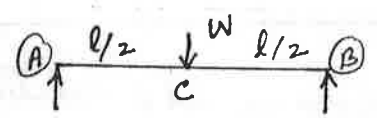
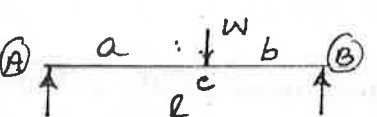
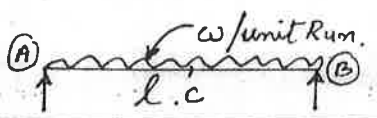
$$\begin{aligned} \rightarrow \text{Max defln } \delta_{\text{max}} &= \frac{W b (a^2 + 2ab)^{3/2}}{9\sqrt{3} E I l} \\ &= \frac{60 \times 10^3 \times 3000 (5000^2 + 2 \times 5000 \times 3000)^{3/2}}{9\sqrt{3} \times 2.1 \times 10^5 \times 1000 \times 10^4 \times 8000} \end{aligned}$$

$$= \underline{280.36 \text{ mm.}}$$

HIGH LIGHTS

① Basic equation to obtain slope and deflection

$$M = EI \frac{d^2 y}{dx^2} \quad (\text{Double integration method}).$$

	BEAM	DEFLECTION	SLOPE
②		$\delta_B = \frac{Wl^3}{3EI}$	$\theta_B = \frac{Wl^2}{2EI}$
③		$\delta_B = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(l-a)$	$\theta_B = \frac{Wa^2}{2EI}$
④		$\delta_B = \frac{wl^4}{8EI}$	$\theta_B = -\frac{wl^3}{6EI}$
⑤		$\delta_B = \frac{wl^4}{30EI}$	$\theta_B = -\frac{wl^3}{24EI}$
⑥		$\delta_B = \frac{wl^4}{8EI} - \frac{wl^4}{30EI}$	
⑦		$\delta_C = \frac{Wl^3}{48EI}$	$\theta_A = \theta_B = -\frac{Wl^2}{16EI}$
⑧		$\delta_C = \frac{Wa^2b^2}{3EI l}$; $\delta_{max} = \frac{Wb(a^2 + 2ab)^{3/2}}{9\sqrt{3} EI \cdot l}$	
⑨		$\delta_C = \frac{5}{384} \frac{wl^4}{EI}$	$\theta_A = \theta_B = -\frac{wl^3}{24EI}$

⑩ Macaulay's method is similar to double integration method. It is convenient for finding deflection in beams subjected to point loads or discontinuous loads.

⑪ Moment area Theorem 1: Slope $\theta = \frac{1}{EI} \times \text{Area of BM diagram}$.

⑫ Moment area Theorem 2: Defl'n $y = \delta = \frac{1}{EI} \times \text{Moment of BM diagram about point of ZERO deflection}$.

SHORT QUESTIONS.

Q1: Write expression for maximum deflection and slope for a cantilever beam of span l , flexural rigidity EI due to a point load W at the free end.

Ans 1: Maximum deflection $\delta = \frac{Wl^3}{3EI}$
 Maximum Slope $\theta = \frac{Wl^2}{2EI}$

Q2: State and explain double integration method.

Ans: Double integration method is used to find slope and deflection in beams.

Basic Equation $EI \frac{d^2y}{dx^2} = M$. Integrating deflection may be calculated $\Rightarrow EI \frac{dy}{dx} = M + C_1$. Again Integrating

$$EI y = Mx + C_1x + C_2$$

C_1 and C_2 may be obtained by applying end conditions.

Q3: Define flexural rigidity and torsional rigidity.

Ans 3: Flexural rigidity $= EI$; Torsional rigidity $= GJ$.

Q4: Find modulus of elasticity 'E' of the material of a simply supported beam of 3m span subjected to central point load 20kN, deflection in beam = 10mm. and $I = 5.6 \times 10^6 \text{ mm}^4$.

Ans 4: $y = \delta = \frac{Wl^3}{48EI} \Rightarrow 10 = \frac{20 \times 10^3 \times (3 \times 10^3)^3}{48 E \times 5.6 \times 10^6}$

$$E = \underline{2.01 \times 10^5 \text{ N/mm}^2}$$

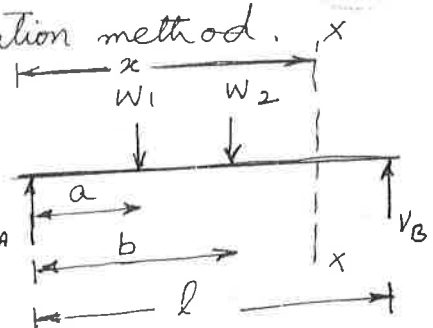
Q5: write short note on Macaulay's method.

Ans5: Macaulay's method is conveniently applied to beams with point loads or discontinuous loads.

It is same as double integration method.

$$EI \frac{d^2 y}{dx^2} = M$$

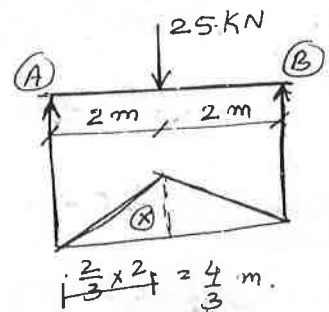
$$= V_A x - \left[W_1 (x-a) \right] - W_2 (x-b) + V_B x$$



Q6: A simply supported beam of 4m span is subjected to central point load of 25 kN. Find deflection at mid span using moment area method.

Ans6: Deflection $y = \delta = \frac{1}{EI}$ (Moment of BM diagram about A)

$$BM \ M = \frac{wl}{4} = \frac{25 \times 4}{4} = 25 \text{ kN-m.}$$



$$y = \frac{1}{EI} \times \left(\frac{1}{2} \times 2 \times 25 \right) \times \frac{4}{3}$$

$$y = \frac{33.33}{EI} \text{ units.}$$

Q7: write a note on Mohr's Theorem to find deflection of beams.

Ans7: Mohr's Theorem is MOMENT AREA METHOD.

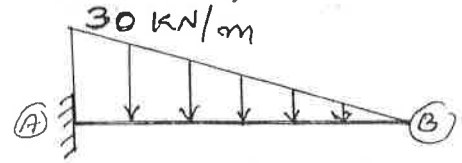
THEOREM 1:

Slope at any point $\theta = \frac{1}{EI}$ (Area of BM diagram)

THEOREM 2:

Deflection at any point $y = \frac{1}{EI}$ (Moment of BM diagram about point of ZERO Deflection)

Q8: A cantilever beam of 4m span is subjected to UVL as shown. Find the maximum deflection.

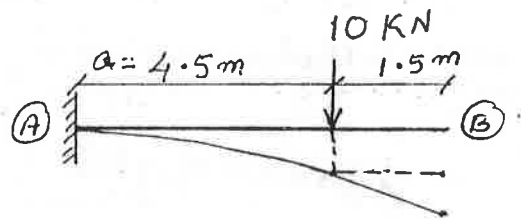


Ans 8: Deflection $\delta_B = \frac{wl^4}{30EI}$

$$= \frac{30 \times 4^4}{30EI}$$

$$= \frac{256}{EI} \text{ units.}$$

Q9: A cantilever beam of 6m is subjected to a point load of 10 kN at a distance of 4.5m from fixed end. Find the deflection at free end.



Ans. 9:

Deflection at B

$$\delta_B = \frac{wa^3}{3EI} + \frac{wa^2}{2EI}(l-a)$$

$$= \frac{10 \times 4.5^3}{3EI} + \frac{10 \times 4.5^2}{2EI}(6 - 4.5)$$

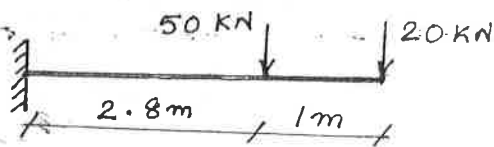
$$\delta_B = \frac{455.62}{EI} \text{ units.}$$

EXERCISE QUESTIONS

1. Derive the differential equation of beams, ^{used} to find the deflection and slope of beams.
2. Explain Macaulay's Method for finding slope and deflection of beams.
3. State and prove Mohr's theorems (Moment area method) to find slope and deflection of beams.
4. A simply supported beam of 9m span is subjected to central point load $W = 83 \text{ kN}$. Calculate the maximum slope and deflection if $E = 2 \times 10^5 \text{ MPa}$ and $I = 20460 \text{ cm}^4$.
(Ans Deflection = 30.80 mm,
Slope = 0.010 radians)

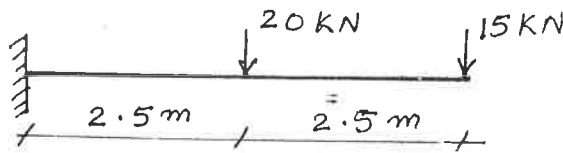
- ⑤ A simply supported ^{beam} of 4.5m span is subjected to a udl of 36 kN/m over its right half. Compute the maximum deflection, $E = 200 \text{ GPa}$ and $I = 100 \times 10^6 \text{ mm}^4$.
(Ans Deflection = 4.8 mm)

- ⑥ A cantilever beam of span 3.8 m carries two point loads as shown in figure. Determine deflection and slope at the free end if $E = 2.05 \times 10^5 \text{ MPa}$ and $I = 5.85 \times 10^8 \text{ mm}^4$



(Ans Deflection = 7.73 mm
Slope = 0.0028 rad)

- ⑦ A cantilever beam of 5m span carries point loads as shown in figure. The maximum deflection in the beam is 13.89 mm and permitted slope is 0.001 rad. Find the value of EI .

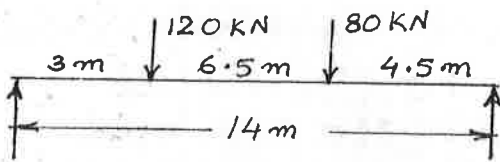


$$\left[\begin{array}{l} \text{Ans} \\ EI = 250 \times 10^{12} \\ \text{N-mm}^2 \end{array} \right]$$

- ⑧ Using moment area method find the slope and deflection in a simply supported beam of span 9m carrying a central point load of 83 kN. $E = 2 \times 10^5 \text{ N/mm}^2$
 $I = 20.46 \times 10^7 \text{ mm}^4$

$$\left[\begin{array}{l} \text{Ans } \delta = 30.80 \text{ mm} \\ \theta = 0.10 \text{ radians} \end{array} \right]$$

- ⑨ Using Macaulay's method find the maximum deflection for the beam shown below. given $E = 2.10 \times 10^5 \text{ N/mm}^2$ and $I = 16 \times 10^8 \text{ mm}^4$.



$$\left[\begin{array}{l} \text{Ans } y_{\text{max}} = 23.6 \text{ mm} \end{array} \right]$$

- ⑩ A simply supported beam of span 'l' is subjected to point load W at centre. Find the deflection under the load using moment area method. Flexural rigidity = EI .

$$\left[\begin{array}{l} \text{Ans } \delta = \frac{Wl^3}{48EI} \end{array} \right]$$

$$e_v = \frac{\sigma \cdot d}{4tE} - \frac{\sigma \cdot d}{m \cdot 2tE} + \frac{\sigma \cdot d}{tE} - \frac{\sigma \cdot d}{m \cdot 2tE}$$

$$= \frac{\sigma \cdot d}{tE} \left(\frac{1}{4} + 1 \right) - \frac{\sigma \cdot d + \sigma \cdot d}{2 \cdot m \cdot t \cdot E}$$

$$= \frac{\sigma \cdot d}{tE} \cdot \frac{5}{4} - \frac{2\sigma \cdot d}{2m \cdot t \cdot E}$$

$$e_v = \frac{\sigma \cdot d}{2tE} \left(\frac{5}{2} - \frac{2}{m} \right)$$

Maximum Shear Stress ν_{max}

$$\nu_{max} = \frac{\text{Hoop stress}}{2}$$

$$= \frac{\sigma_H}{2}$$

$$= \frac{\sigma \cdot d}{2 \times 2t}$$

$$\nu_{max} = \frac{\sigma d}{4t}$$

THIN SPHERICAL SHELLS :

Figure shows a thin spherical shell

d = Internal dia of shell.

t = Thickness of wall

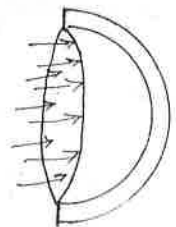
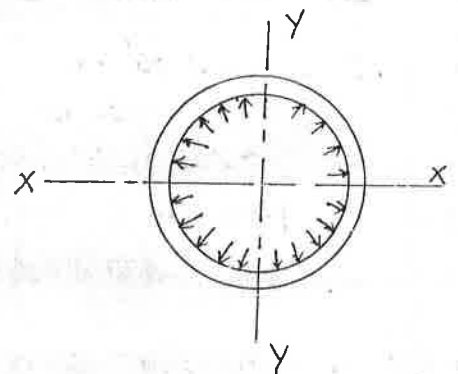
σ = Internal pressure intensity.

Consider section $x-x$ bisecting the sphere

$$\text{Bursting force } P_B = \sigma \times \text{projected area.}$$

$$= \sigma \times \frac{\pi d^2}{4}$$

$$\text{Resisting force} = \sigma_1 \times \pi d t \quad (\sigma_1 = \text{Resisting stress})$$



(4)

prob 1:

A seamless thin cylindrical pipe of 1.25 m internal diameter is to carry fluid under a pressure of 2.25 N/mm^2 . Calculate the thickness of pipe required if maximum permissible stress is 87.5 N/mm^2 .

SOLUTION

Given $d = 1.25 \text{ m} = 1.25 \times 10^3 \text{ mm}$ or $r = 0.625 \times 10^3 \text{ mm}$.

$$\sigma = 2.25 \text{ N/mm}^2$$

permissible stress $\sigma_L = \sigma_H = 87.5 \text{ N/mm}^2$.

Thickness from Hoop Stress Consideration:

$$\rightarrow \text{Max Hoop stress } \sigma_H = \frac{\sigma \cdot d}{2t}$$

$$87.5 = \frac{2.25 \times 1.25 \times 10^3}{2t}$$

$$\text{Thickness } t = \underline{\underline{16.07 \text{ mm}}}$$

Thickness for Longitudinal Stress Consideration:

$$\rightarrow \text{Max. Longitudinal stress } \sigma_L = \frac{\sigma d}{4t}$$

$$87.05 = \frac{2.25 \times 1.25 \times 10^3}{4t}$$

$$t = \underline{\underline{8.035 \text{ mm}}}$$

provide thickness $\boxed{t = 16.07 \text{ mm}}$ from Hoop Stress consideration.

Prob 2:

A cylindrical container 3.30 m long, 1.25 m internal dia has 16 mm metal thickness. It is subjected to an internal fluid pressure of 2 N/mm^2 . Taking $E = 2.0 \times 10^5 \text{ N/mm}^2$ Calculate

(a) Maximum Hoop stress σ_H .

(b) Maximum Longitudinal stress σ_L .

(c) Maximum Shear stress $\tau_{\max}$.

SOLUTION:

$$L = 3.30 \text{ m} = 3.30 \times 10^3 \text{ mm.}$$

$$d = 1.25 \text{ m} = 1.25 \times 10^3 \text{ mm} \Rightarrow r = 0.625 \times 10^3 \text{ mm.}$$

$$t = 16 \text{ mm} \quad ; \quad \sigma = 2 \text{ N/mm}^2.$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\begin{aligned} \text{(a) Maximum Hoop stress } \sigma_H &= \frac{\sigma \cdot d}{2t} \\ &= \frac{2 \times 1.25 \times 10^3}{2 \times 16} \\ &= \underline{\underline{78.125 \text{ N/mm}^2}} \end{aligned}$$

$$\begin{aligned} \text{(b) Maximum Longitudinal stress } \sigma_L &= \frac{\sigma \cdot d}{4t} \\ &= \frac{2 \times 1.25 \times 10^3}{4 \times 16} \\ &= \underline{\underline{39.06 \text{ N/mm}^2}} \end{aligned}$$

$$\begin{aligned} \text{(c) Maximum Shear Stress } \tau_{\max} &= \frac{\sigma \cdot d}{4t} \\ &= \underline{\underline{39.06 \text{ N/mm}^2}} \end{aligned}$$

Prob 3:

A thin cylindrical vessel store fluid at an internal stress of 3.2 N/mm^2 . Length of the vessel is 4 m and its dia is 1.3 m , thickness of wall of the vessel is 18 mm . Taking $E = 2.05 \text{ N/mm}^2$ and $\frac{1}{m} = 0.33$ Find

(a) Circumferential strain e_c .

(b) Longitudinal strain e_L .

(c) Volumetric strain e_v .

Solution:

$$\sigma = 3.2 \text{ N/mm}^2 ; L = 4 \text{ m} = 4 \times 10^3 \text{ mm}$$

$$\text{dia } d = 1.3 \text{ m} = 1.3 \times 10^3 \text{ mm} \Rightarrow r = 0.65 \times 10^3 \text{ mm}$$

$$t = 18 \text{ mm} , E = 2.05 \text{ N/mm}^2 ; \frac{1}{m} = 0.33$$

$$\text{(a) Circumferential strain } e_c = \frac{\sigma_H}{E} - \frac{\sigma_L}{mE} \quad \text{--- (1)}$$

$$\Rightarrow \sigma_H = \frac{\sigma d}{2t} = \frac{3.2 \times 1.3 \times 10^3}{2 \times 18} = 115.55 \text{ N/mm}^2 \checkmark$$

$$\sigma_L = \frac{\sigma d}{4t} = \frac{3.2 \times 1.3 \times 10^3}{4 \times 18} = 57.78 \text{ N/mm}^2 \checkmark$$

Substituting σ_H and σ_L in Eqn (1).

$$\begin{aligned} \text{Circumferential strain } e_c &= \frac{115.55}{2.05 \times 10^5} - \frac{0.3 \times 57.78}{2.05 \times 10^5} \\ &= 5.64 \times 10^{-4} - 8.45 \times 10^{-5} \end{aligned}$$

$$\boxed{e_c = 4.79 \times 10^{-4}}$$

(b) Longitudinal strain $e_L = \frac{\sigma_L}{E} - \frac{1}{m} \frac{\sigma_H}{E}$

$$e_L = \frac{57.78}{2.05 \times 10^5} - 0.3 \times \frac{115.55}{2.05 \times 10^5}$$

$$= 1.13 \times 10^{-4}$$

(c) Volumetric strain $e_v = e_L + 2e_c$

$$e_v = 1.13 \times 10^{-4} + 2 \times 4.79 \times 10^{-4}$$

$$e_v = 1.071 \times 10^{-3}$$

prob 4! JNTU May/June 2009 Suppl. Set No 1

(a) Derive from first principles the expression for circumferential and longitudinal stresses in a thin cylinder closed at both ends and subjected to internal fluid pressure.

(b) What thickness of metal would be required for cast-iron water pipe 80cm in diameter under a head of 100m? Assume the permissible tensile stress for cast-iron as 20 MN/m^2 .

SOLUTION:

(a) Refer notes for derivation.

(b) dia of pipe $d = 80 \text{ cm} = 0.8 \text{ m}$.

Head of water $H = 100 \text{ m}$.

$$\text{permissible tensile stress} = 20 \text{ MN/m}^2 = 20 \times \frac{10}{10^3 \times 10^3} \text{ N/mm}^2$$

$$= 20 \text{ N/mm}^2$$

pressure developed due to water head

σ = wt of water distributed on circumference of pipe.

$$= \frac{\text{volume} \times \text{density}}{(\pi d) t}$$

$$= \frac{\frac{\pi \times d^2}{4} \times H \times 1000}{(\pi d) t} = \text{Kg/m}^2 \quad \left(\text{density of water} = 1000 \text{ Kg/m}^3 \right)$$

$$= \frac{d \times H}{4t} \times 1000$$

$$= \frac{0.8 \times 100}{4t} \times 1000$$

$$= \frac{20,000}{t} \text{ Kg/m}^2$$

$$= \frac{20,000}{t} \times \frac{10}{10^3 \times 10^3}$$

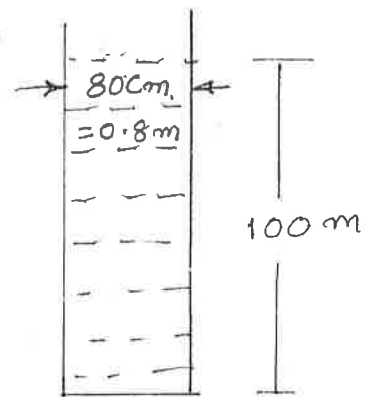
$$\sigma = \frac{2}{t} \text{ N/mm}^2$$

→ Hoop stress developed σ_H .

$$\sigma_H = \frac{\sigma d}{2t}$$

$$20 = \frac{2 \times 80 \times 10}{t(2 \times t)}$$

$$t = 6.32 \text{ mm.}$$



Prob 5: JNTU May/June 2009 Suppl. Set No (2).

- (a) Derive the expressions for change in diameter and volume of a thin spherical shell due to an internal pressure.
- (b) A spherical shell 3m in dia is subjected to an internal pressure of 2 N/mm^2 . Find the thickness of the plate required if maximum stress is not to exceed 80 N/mm^2 . Take efficiency of joint as 75%.

SOLUTION

(a) Refer notes for derivation.

(b) dia $d = 3 \text{ m} = 3 \times 10^3 \text{ mm}$; $\sigma = 2 \text{ N/mm}^2$

$$\sigma_1 = 80 \text{ N/mm}^2 ; \eta = 75\% = \frac{75}{100} = 0.75$$

$$\text{Resisting stress } \sigma_1 = \frac{\sigma \cdot d}{4t \cdot \eta}$$

$$80 = \frac{2 \times 3 \times 10^3}{4 \times t \times 0.75}$$

Thickness of plate $t = 25 \text{ mm.}$ ✓

prob 6: JNTU May/June 2009 ⁽¹³⁾ Set NE (2)

The gauge pressure in a boiler 1.5 m dia and 12.5 mm thickness is 2 MN/m^2 , find the longitudinal and circumferential stress in the boiler plate, if the boiler was built up and the longitudinal and circumferential efficiencies of the joints were 80% and 50% respectively. Also find the circumferential and longitudinal strains. Take $E = 200 \text{ GN/m}^2$ and $\frac{1}{m} = 0.25$.

SOLUTION:

$$d = 1.5 \text{ m} = 1.5 \times 10^3 \text{ mm} ; t = 12.5 \text{ mm} ; \sigma = 2 \text{ MN/mm}^2 = \frac{2 \times 10^6}{10^6} = 2 \text{ N/mm}^2$$
$$\eta_L = 80\% = \frac{80}{100} = 0.80 ; \eta_c = 50\% = \frac{50}{100} = 0.50.$$

$$\sigma_L = ? \quad e_L = ? \quad \sigma_H = ? , e_c = ?$$

$$\rightarrow \text{Circumferential stress} = \text{Hoop stress} = \sigma_H.$$

$$\rightarrow \text{Circumferential strain} = \text{Hoop strain} = e_c$$

$$\rightarrow \text{Longitudinal stress } \sigma_L = \frac{\sigma d}{4t \eta_L} = \frac{2 \times 1.5 \times 10^3}{4 \times 12.5 \times 0.80}$$
$$= 75 \text{ N/mm}^2.$$

$$\rightarrow \text{Circumferential stress or Hoop stress } \sigma_H = \frac{\sigma d}{2t \eta_c} = \frac{2 \times 1.5 \times 10^3}{2 \times 12.5 \times 0.5}$$
$$= \underline{\underline{240 \text{ N/mm}^2}}$$

$$\rightarrow \text{Longitudinal strain } e_L = \frac{\sigma_L}{E} - \frac{1}{m} \frac{\sigma_H}{E} = \frac{75}{2 \times 10^5} - \frac{240 \times 0.25}{2 \times 10^5}$$
$$= \underline{\underline{7.5 \times 10^{-5}}}$$

$$\rightarrow \text{Circumferential strain } e_c = e_H = \frac{\sigma_H}{E} - \frac{1}{m} \frac{\sigma_L}{E} = \frac{240}{2 \times 10^5} - \frac{0.25 \times 75}{2 \times 10^5}$$
$$= \underline{\underline{1.10 \times 10^{-3}}}$$

- (a) Derive an expression for circumferential stress and stress for a thin shell subjected to an internal pressure.
- (b) A cylindrical vessel whose ends are closed by means of rigid plates is made of steel plate 3mm thick. The internal length and dia of vessel are 50cm, and 25cm respectively. Determine the longitudinal and circumferential stresses in the cylindrical shell due to an internal fluid pressure of 3 MN/m^2 .

SOLUTION

(a) Refer notes for derivation.

(b) $t = 3 \text{ mm}$. $l = 50 \text{ cm} = 500 \text{ mm}$. ; $L = 25 \text{ cm} = 250 \text{ mm}$.

$$\sigma = 3 \text{ MN/m}^2 = 3 \times \frac{10^6}{10^3 \times 10^3} = 3 \text{ N/mm}^2$$

$$\sigma_L = ? \quad \sigma_H = ?$$

$$\begin{aligned} \rightarrow \text{Longitudinal Stress } \sigma_L &= \frac{\sigma \cdot d}{4t} \\ &= \frac{3 \times 250}{4 \times 3} \\ \sigma_L &= \underline{\underline{62.5 \text{ N/mm}^2}} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{Circumferential stress} \} \sigma_H &= \frac{\sigma \cdot d}{2t} \\ \text{or Hoop Stress.} &= \frac{3 \times 250}{2 \times 3} \\ \sigma_L &= \underline{\underline{125 \text{ N/mm}^2}} \end{aligned}$$

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Prob 8: JNTU May/June 2009. Suppl. Set No (3) Code: RR 210102

A shell 3.25m long, 1m in dia is subjected to an internal pressure of 1 N/mm^2 . If the thickness of the shell is 10mm, find the circumferential and longitudinal stresses. Also find the maximum shear stress and change in the dimensions of the shell.

Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\mu = \frac{1}{m} = 0.3$.

SOLUTION: $l = 3250 \text{ mm}$; $d = 1000 \text{ mm}$; $\sigma = 1 \text{ N/mm}^2$
 $t = 10 \text{ mm}$; $E = 2 \times 10^5 \text{ N/mm}^2$, $\frac{1}{m} = 0.3$.

→ Circumferential stress } $\sigma_H = \frac{\sigma d}{2t} = \frac{1 \times 1000}{2 \times 10} = 50 \text{ N/mm}^2$
or Hoop stress

→ Longitudinal stress $\sigma_L = \frac{\sigma d}{4t} = \frac{1 \times 1000}{4 \times 10} = 25 \text{ N/mm}^2$.

→ Max. Shear stress $\tau_{\max} = \frac{\sigma d}{4t} = \frac{1 \times 1000}{4 \times 10} = 25 \text{ N/mm}^2$

→ Circumferential strain $\frac{\delta d}{d} = e_v = \frac{\sigma_H}{E} - \frac{1}{m} \frac{\sigma_L}{E} =$
 $= \frac{50}{2 \times 10^5} - \frac{0.3 \times 25}{2 \times 10^5}$
 $= 2.125 \times 10^{-4}$
 $\frac{\delta d}{1000} = 2.125 \times 10^{-4}$

Change in diameter $\boxed{\delta d = 0.212 \text{ mm.}}$

→ Longitudinal strain $e_L = \frac{\delta L}{L} = \frac{\sigma_L}{E} - \frac{1}{m} \frac{\sigma_H}{E}$
 $= \frac{25}{2 \times 10^5} - \frac{0.3 \times 50}{2 \times 10^5}$
 $= 5 \times 10^{-5}$

Change in Length $\frac{\delta L}{3250} = 5 \times 10^{-5}$
 $\boxed{\delta L = 0.162 \text{ mm}}$

unit VI PRINCIPAL STRESSES AND STRAINS

Introduction:

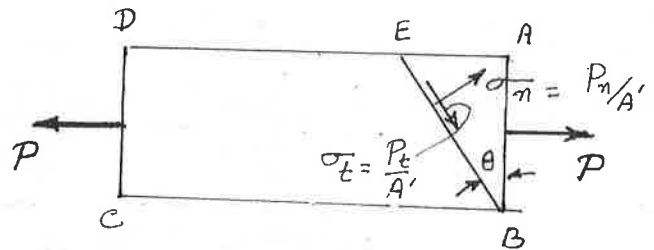
In unit I and unit II we studied the behaviour of members subjected to tension, compression or shear stresses. In actual practice all the three types of stresses may act simultaneously on some planes passing through a point in the strained material.

It may be noted that some inclined planes may carry greater forces than the applied one, due to combination of stresses. On some planes only NORMAL stresses exist, these planes are called **PRINCIPAL STRESSES** and the planes are called **PRINCIPAL PLANES**.

Stresses on an Inclined Section of a bar

under Axial Load:

Consider the bar shown in figure subjected to axial load 'P'.



Let A = Area of cross section at AB .

Consider an inclined section BE making an angle θ with AB .

Let A' = Area of cross section at BE .

$$\text{From figure } \cos \theta = \frac{AB}{BE} \Rightarrow BE = AB \sec \theta \Rightarrow \boxed{A' = A \sec \theta}$$

(2)

Given P = Direct tensile (axial) force

$$\text{Direct Stress on AC} \left\{ \sigma_1 = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} \right.$$

$$\rightarrow \text{Force normal to BE } P_n = P \cos \theta.$$

$$\rightarrow \text{Stress normal to BE, } \sigma_n = \frac{P_n}{A'} = \frac{P \cos \theta}{A \sec \theta}$$

$$\rightarrow \boxed{\sigma_n = \sigma_1 \cos^2 \theta.}$$

$$\begin{aligned} \rightarrow \text{Stress tangential to BE, } \sigma_t &= \frac{P_t}{A'} \\ &= \frac{P \sin \theta}{A \sec \theta} \\ &= \sigma_1 \sin \theta \cos \theta \end{aligned}$$

$$\rightarrow \boxed{\sigma_t = \frac{\sigma_1}{2} \sin 2\theta}$$

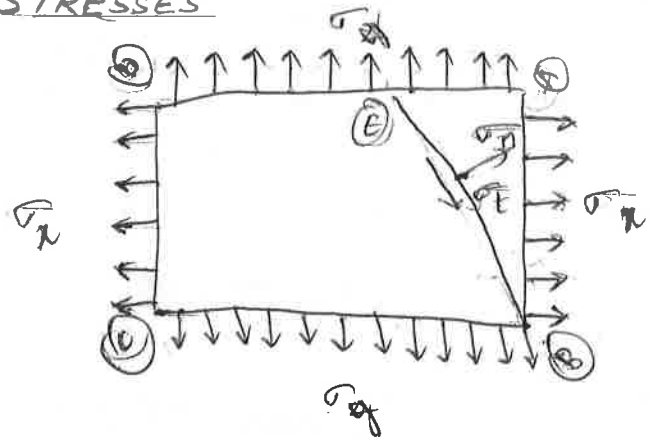
$$\begin{aligned} \rightarrow \text{Resultant stress } \sigma_R &= \sqrt{\sigma_n^2 + \sigma_t^2} \\ &= \sqrt{(\sigma_1 \cos^2 \theta)^2 + \left(\frac{\sigma_1}{2} \sin 2\theta\right)^2} \\ &= \sigma_1 \sqrt{\cos^4 \theta + \left(\frac{2 \sin \theta \cos \theta}{2}\right)^2} \\ &= \sigma_1 \sqrt{\cos^4 \theta + \sin^2 \theta \cos^2 \theta} \\ &= \sigma_1 \sqrt{\cos^2 \theta (\cos^2 \theta + \sin^2 \theta)} \end{aligned}$$

$$\rightarrow \boxed{\sigma_R = \sigma_1 \cos \theta.}$$

Normal and Tangential Stresses on an Inclined plane for BIAXIAL STRESSES

Consider the block ABCD shown in figure

Let thickness of block perpendicular to plane of paper be unity.



σ_1 = stress along x-axis

σ_2 = stress along y-axis.

Consider an oblique plane BE at angle θ with AB.

→ Normal stress on the plane BE σ_n

$$\sigma_n = \frac{\text{Total force normal to the plane BE}}{\text{Sectional area along BE}}$$

$$= \frac{(\sigma_1 \times BA) \cos \theta + (\sigma_2 \times EA) \sin \theta}{BE}$$

$$= \sigma_1 \frac{BA}{BE} \cos \theta + \sigma_2 \frac{EA}{BE} \sin \theta$$

$$= \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta. \quad \text{--- (1)}$$

$$= \frac{\sigma_1}{2} (1 + \cos 2\theta) + \frac{\sigma_2}{2} (1 - \cos 2\theta)$$

$$= \frac{\sigma_1}{2} + \frac{\sigma_1}{2} \cos 2\theta + \frac{\sigma_2}{2} - \frac{\sigma_2}{2} \cos 2\theta$$

$$\therefore \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta. \quad \text{--- (2)}$$

(Note:
Area = BE × 1
= BE
Since thickness
is 1 unit)

(4)

→ Tangential stress on the plane BE, σ_t

$$\sigma_t = \frac{\text{Total tangential force on BE}}{\text{Sectional area along BE.}}$$

$$= \frac{(\sigma_1 \cdot BA) \sin \theta - \sigma_2 EA \cos \theta}{BE \times 1}$$

$$= \sigma_1 \cdot \frac{BA}{BE} \cdot \sin \theta - \sigma_2 \frac{EA}{BE} \cos \theta$$

$$= \sigma_1 \cos \theta \sin \theta - \sigma_2 \sin \theta \cos \theta$$

$$= (\sigma_1 - \sigma_2) \sin \theta \cos \theta. \quad \text{--- (3)}$$

$$\Rightarrow = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta. \quad \text{--- (4)}$$

→ Resultant stress on BE is the resultant of σ_n and σ_t

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} \quad \text{From Eqn ① and ③.}$$

$$= \sqrt{(\sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta)^2 + (\sigma_1 - \sigma_2)^2 \sin^2 \theta \cos^2 \theta}$$

$$= \sqrt{\sigma_1^2 \cos^4 \theta + \sigma_2^2 \sin^4 \theta + 2\sigma_1 \sigma_2 \sin^2 \theta \cos^2 \theta + \sigma_1^2 \sin^2 \theta \cos^2 \theta + \sigma_2^2 \sin^2 \theta \cos^2 \theta - 2\sigma_1 \sigma_2 \sin^2 \theta \cos^2 \theta}$$

$$= \sqrt{\sigma_1^2 \cos^2 \theta (\cos^2 \theta + \sin^2 \theta) + \sigma_2^2 \sin^2 \theta (\sin^2 \theta + \cos^2 \theta)}$$

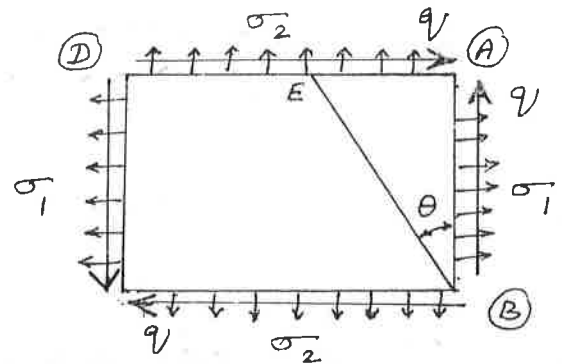
$$\Rightarrow = \sqrt{\sigma_1^2 \cos^2 \theta + \sigma_2^2 \sin^2 \theta}$$

→ Let Φ = Angle made by resultant with normal

$$\Rightarrow \tan \Phi = \frac{\sigma_t}{\sigma_n}$$

Two perpendicular Normal Stresses Occompanied by a state of Simple shear :

Consider block ABCD.
Thickness of block is
unit (1mm) perpendicular
to the plane of paper.



σ_1 and σ_2 are normal stresses along x and y - axes.
 q is shear stress acting on the block.

→ Normal stress on the plane BE, σ_n :

$$\sigma_n = \frac{(\sigma_1 \cdot AB) \cos \theta + (\sigma_2 \cdot AE) \sin \theta + q \cdot AE \cos \theta + q \cdot AB \sin \theta}{BE \times 1}$$

$$= \sigma_1 \cdot \frac{AB}{BE} \cdot \cos \theta + \sigma_2 \cdot \frac{AE}{BE} \sin \theta + q \cdot \frac{AE}{BE} \cos \theta + q \cdot \frac{AB}{BE} \sin \theta$$

$$= \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta + q \sin \theta \cos \theta + q \cos \theta \sin \theta$$

$$= \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta + 2q \sin \theta \cos \theta$$

$$= \frac{\sigma_1}{2} (1 + \cos 2\theta) + \frac{\sigma_2}{2} (1 - \cos 2\theta) + q \cdot \sin 2\theta$$

$$\rightarrow \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + q \cdot \sin 2\theta$$

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→ Tangential Stress on plane BE, σ_t :

$$\sigma_t = \frac{(\sigma_1 \cdot AB) \sin \theta - (\sigma_2 \cdot AE) \cos \theta + (q \cdot AE) \sin \theta - (q \cdot AB) \cos \theta}{BE \times 1}$$

$$= \sigma_1 \cdot \frac{AB}{BE} \sin \theta - \sigma_2 \cdot \frac{AE}{BE} \cos \theta + q \cdot \frac{AE}{BE} \sin \theta - q \cdot \frac{AB}{BE} \cos \theta$$

$$= \sigma_1 \cos \theta \sin \theta - \sigma_2 \sin \theta \cos \theta + q \sin^2 \theta - q \cos^2 \theta$$

$$\rightarrow \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - q \cos 2\theta$$

→ For the plane to be principal plane, the tangential stress σ_t must be ZERO.

Equating σ_t to ZERO.

$$\frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - q \cos 2\theta = 0$$

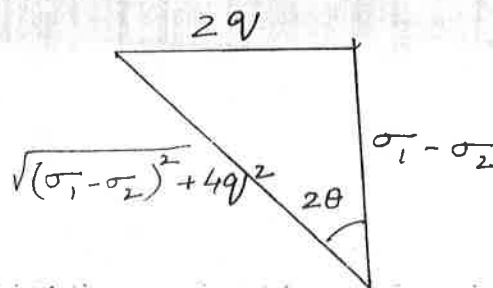
$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2q}{\sigma_1 - \sigma_2}$$

$$\boxed{\tan 2\theta = \frac{2q}{\sigma_1 - \sigma_2}}$$

→ There exist two values of 2θ differing by 180° satisfying the above equation.

Let $2\theta_1$ and $2\theta_2$ be the solutions

$$\sin 2\theta_1 = \frac{2q}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}} \quad \text{and} \quad \sin 2\theta_2 = \frac{-2q}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}}$$



$$\cos 2\theta = \frac{(\sigma_1 - \sigma_2)}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}} \quad \text{and} \quad \cos 2\theta_2 = \frac{(\sigma_1 - \sigma_2)}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

→ The values of θ_1 and θ_2 differ by 90° . Therefore the principal planes are normal to each other.

Substituting $2\theta_1$ and $2\theta_2$ in Normal Stress: σ_n
Principal stresses σ_{p_1} and σ_{p_2} may be calculated.

→ MAJOR PRINCIPAL STRESS:

$$\sigma_{p_1} = \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cos 2\theta_1 + \tau \sin 2\theta_1$$

$$= \frac{(\sigma_1 + \sigma_2)}{2} + \frac{(\sigma_1 - \sigma_2)}{2} \cdot \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}} + \frac{2\tau^2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{(\sigma_1 - \sigma_2)^2 + 4\tau^2}{2\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$\rightarrow = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} \quad \left(\text{MAJOR PRINCIPAL STRESS} \right)$$

→ MINOR PRINCIPAL STRESS:

$$\sigma_{p_2} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta_2 + \tau \sin 2\theta_2$$

$$= \frac{\sigma_1 + \sigma_2}{2} - \frac{(\sigma_1 - \sigma_2)}{2} \frac{(\sigma_1 - \sigma_2)}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}} - \frac{2\tau^2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$= \frac{\sigma_1 + \sigma_2}{2} - \frac{(\sigma_1 - \sigma_2)^2 + 4\tau^2}{2\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

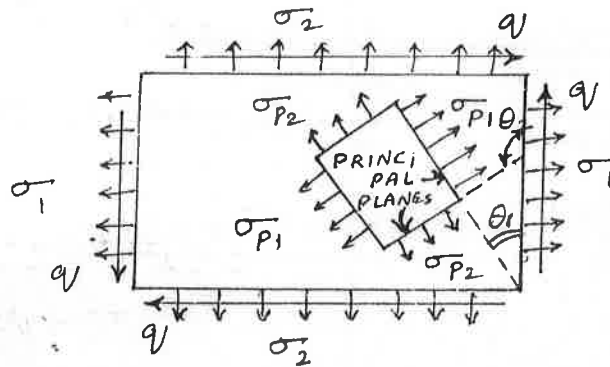
$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} \quad \left(\text{MINOR PRINCIPAL STRESS} \right)$$

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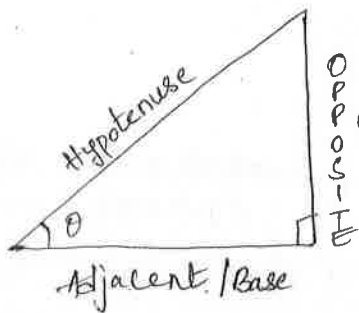
→ Maximum Shear Stress $\tau_{\max}$ occurs at $\theta, +45^\circ$ and $\theta, +135^\circ$.

$$\text{Maximum Shear Stress } \tau_{\max} = \frac{\sigma_{P1} - \sigma_{P2}}{2}$$

→ principal planes are shown in figure below



Note :-



$$\tan \theta = \frac{\text{opposite}}{\text{Adjacent}}$$

$$\sin \theta = \frac{\text{opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Some basic Identity of Trigonometry

$$1) \sin \theta = \frac{1}{\csc \theta}, \csc \theta = \frac{1}{\sin \theta}$$

$$2) \cos \theta = \frac{1}{\sec \theta}, \sec \theta = \frac{1}{\cos \theta}$$

$$3) \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$4) \tan \theta = \frac{1}{\cot \theta}, \cot \theta = \frac{1}{\tan \theta}$$

$$5) \sin^2 \theta + \cos^2 \theta = 1$$

$$\rightarrow \sin^2 \theta = 1 - \cos^2 \theta, \cos^2 \theta = 1 - \sin^2 \theta$$

$$6) \sec^2 \theta - \tan^2 \theta = 1$$

$$\rightarrow \sec^2 \theta = 1 + \tan^2 \theta, \tan^2 \theta = \sec^2 \theta - 1$$

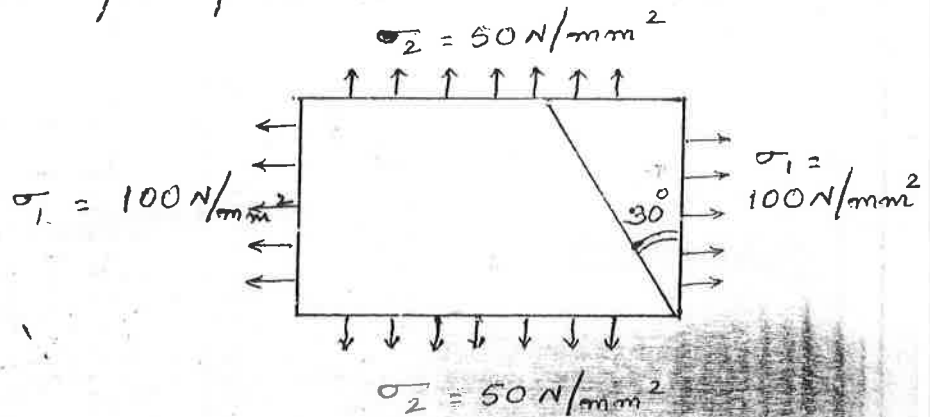
$$7) \csc^2 \theta - \cot^2 \theta = 1$$

$$\rightarrow \csc^2 \theta = 1 + \cot^2 \theta, \cot^2 \theta = \csc^2 \theta - 1$$

Prob 1:

Tensile stresses at a point across two perpendicular planes are 100 N/mm^2 and 50 N/mm^2 . Find the normal and tangential stresses and the resultant stress and its inclination on a plane at 30° with the major principal plane.

Solution:



$$\begin{aligned} \text{Normal Stress } \sigma_n &= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta \\ &= \frac{100 + 50}{2} + \frac{100 - 50}{2} \cos 60^\circ \\ &= \underline{87.5 \text{ N/mm}^2} \end{aligned}$$

$$\begin{aligned} \text{Tangential Stress } \sigma_t &= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta \\ &= \frac{100 - 50}{2} \sin 60^\circ \\ &= \underline{21.65 \text{ N/mm}^2} \end{aligned}$$

$$\begin{aligned} \text{Resultant Stress } \sigma_R &= \sqrt{\sigma_n^2 + \sigma_t^2} \\ &= \sqrt{87.5^2 + 21.65^2} \\ &= \underline{90.13 \text{ N/mm}^2} \end{aligned}$$

$$\text{Inclination } \tan \phi = \frac{\sigma_t}{\sigma_n} \Rightarrow \phi = \tan^{-1} \frac{21.65}{87.50} \Rightarrow \boxed{\phi = 13.9^\circ}$$

Prob 2 : u/v

(10)

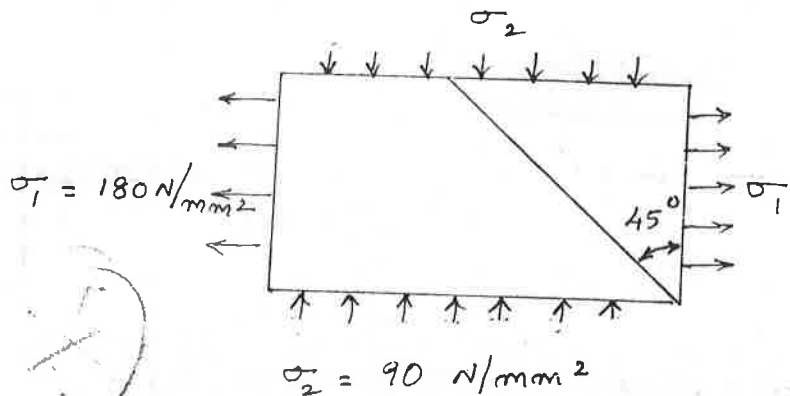
The stresses at a point in a bar are 180 N/mm^2 (Tensile) and 90 N/mm^2 (compressive). Determine the magnitude and direction of resultant stress on a plane inclined at 45° to the axis of major stress. Determine also the magnitude of maximum shear stress.

SOLUTION

$$\sigma_1 = 180 \text{ N/mm}^2$$

$$\sigma_2 = 90 \text{ N/mm}^2$$

$$\theta = 45^\circ$$



→ Normal stress on the plane $\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$.

$$\begin{aligned}\sigma_n &= \frac{180 + (-90)}{2} + \frac{180 - (-90)}{2} \cos 60^\circ \\ &= \underline{\underline{112.5 \text{ N/mm}^2}}\end{aligned}$$

→ Tangential stress on the plane $\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$.

$$\begin{aligned}\sigma_t &= \frac{180 - (-90)}{2} \sin 60^\circ \\ &= \underline{\underline{116.91 \text{ N/mm}^2}}\end{aligned}$$

→ Resultant stress $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

$$\begin{aligned}&= \sqrt{112.5^2 + 116.91^2} \\ &= \underline{\underline{162.25 \text{ N/mm}^2}}\end{aligned}$$

→ Maximum Shear Stress $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{180 - (-90)}{2}$

$$= \underline{\underline{135 \text{ N/mm}^2}}$$

Prob 3: ✕

Prove that the sum of normal components (σ_n) of stress on any two mutually perpendicular planes, is constant if a material is subjected to two-dimensional stress.

Solution

Let σ_1 and σ_2 are stresses

Let θ be the angle with major principal plane

$$\rightarrow \text{Normal stress } \sigma_{n-1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta. \quad (1)$$

$\rightarrow$ Normal stress on other plane $\Rightarrow \theta = (\theta + 90)$

$$\sigma_{n-2} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos(2\theta + 180)$$

$$= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} (-\cos 2\theta)$$

$$= \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta \quad (2)$$

Adding (1) and (2)

$$\sigma_{n-1} + \sigma_{n-2} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}$$

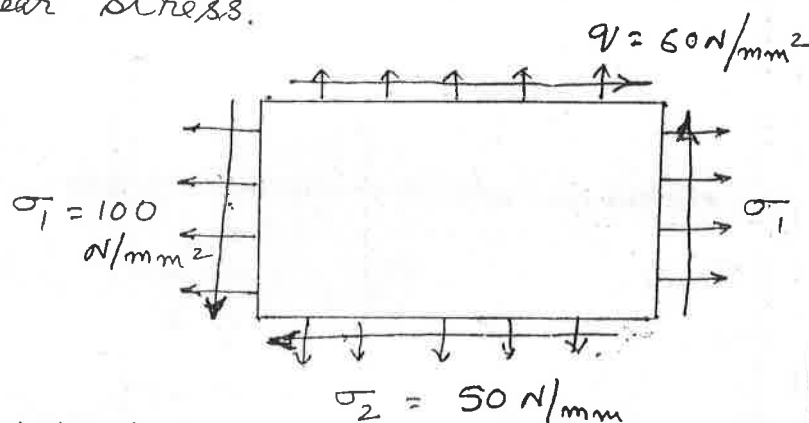
$$= \sigma_1 + \sigma_2 \quad \text{which is a Constant.}$$

Prob 4:

A rectangular block is subjected to a tensile stress of 100 N/mm^2 on one plane and a tensile stress of 50 N/mm^2 of perpendicular plane, alongwith shear stresses of 60 N/mm^2 on the same planes. Determine

- Direction of principal planes.
- Magnitude of principal stress.
- Maximum shear stress.

SOLUTION



① Direction of principal planes.

$$\tan 2\theta = \frac{2q}{\sigma_1 - \sigma_2} = \frac{2 \times 60}{100 - 50} = \underline{\underline{2.4}}$$

$$\theta = \underline{\underline{33.69^\circ}}$$

② Major principal stress $\sigma_{p1} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$

$$\begin{aligned} \sigma_{p1} &= \frac{100 + 50}{2} + \sqrt{\left(\frac{100 - 50}{2}\right)^2 + 60^2} \\ &= \underline{\underline{140 \text{ N/mm}^2}} \end{aligned}$$

$$\begin{aligned} \text{Minor principal stress } \sigma_{p2} &= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2} \\ &= \frac{100 + 50}{2} - \sqrt{\left(\frac{100 - 50}{2}\right)^2 + 60^2} \\ &= \underline{\underline{10 \text{ N/mm}^2}} \end{aligned}$$

③ Maximum Shear Stress $q_{\max} = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \frac{140 - 10}{2} = \underline{\underline{65 \text{ N/mm}^2}}$

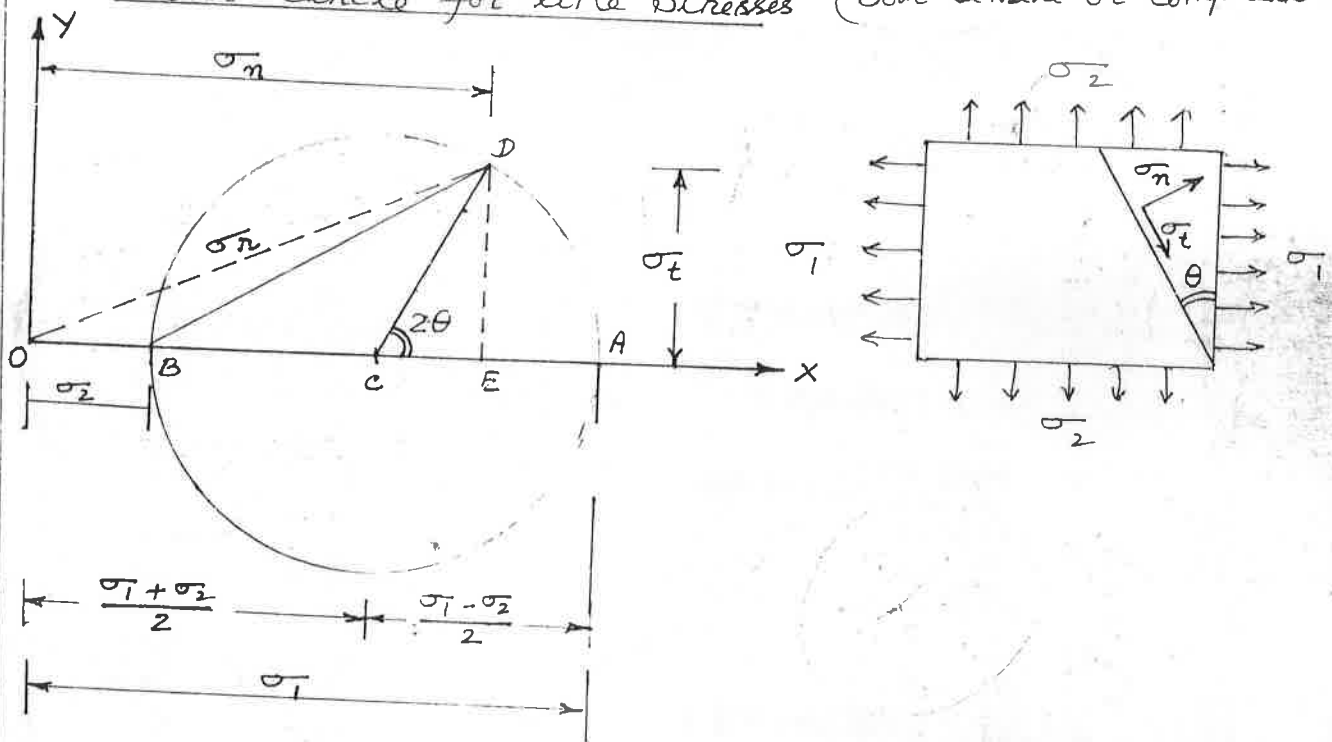
Graphical Solution Stresses

In previous articles Analytical methods were used to determine normal stresses, tangential stresses etc.

In this article graphical solution is studied.

→ Graphical solution suggested by RANKIN is known as MOHR'S CIRCLE or STRESS CIRCLE.

Mohr Circle for like Stresses (Both Tensile or Compressive)



PROCEDURE

- On axis OX draw $OA = \sigma_1$ and $OB = \sigma_2$
- Bisect AB at C. with centre C and radius CA or CB draw a circle.
- At C draw CD at an angle 2θ with CA. CD is drawn in the same direction as the normal to the plane makes with σ_1 .

→ OE gives the norm: stress σ_n .

DE gives the tangential stress σ_t .

OD gives the resultant stress σ_r .

proof:

NORMAL STRESS σ_n

$$\begin{aligned}
 \rightarrow OE &= OC + CE \\
 &= OC + CD \cos 2\theta \\
 &= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos \theta \\
 &= \sigma_n.
 \end{aligned}$$

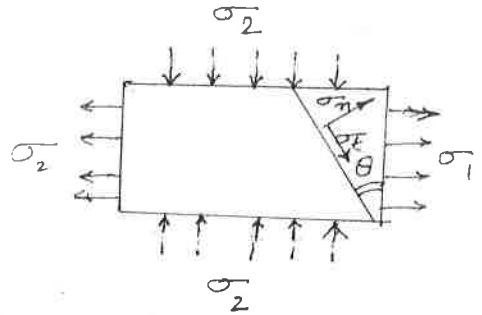
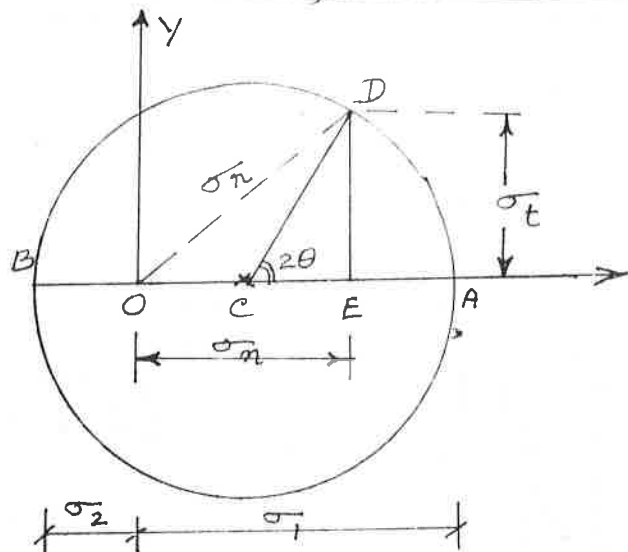
→ Tangential stress σ_t

$$\begin{aligned}
 DE &= CD \sin 2\theta \\
 &= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta \\
 &= \sigma_t.
 \end{aligned}$$

→ Resultant stress σ_r

$$\begin{aligned}
 OD &= \sqrt{OE^2 + DE^2} \\
 &= \sqrt{\sigma_n^2 + \sigma_t^2} \\
 &= \sigma_r.
 \end{aligned}$$

Mohr's Circle for Unlike Stresses:



PROCEDURE

- On axis OX draw $OA = \sigma_1$ and $OB = \sigma_2$. OA and OB are taken in opposite direction since the stresses are unlike (one tensile and one compressive)
- Follow the same steps given in previous solution.
- OE gives normal stress (σ_n), DE gives tangential stress (σ_t) and OD gives resultant stress (σ_n).

Proof:

- Normal stress σ_n ; $OE = OC + CE$

$$= OC + CD \cos 2\theta$$

$$= \frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta$$

$$= \sigma_n.$$
- Tangential stress σ_t ; $DE = CD \sin 2\theta$

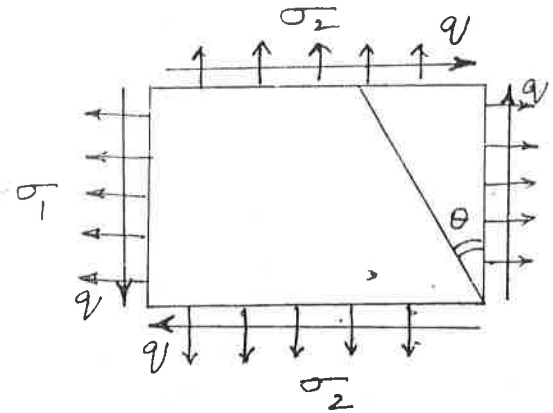
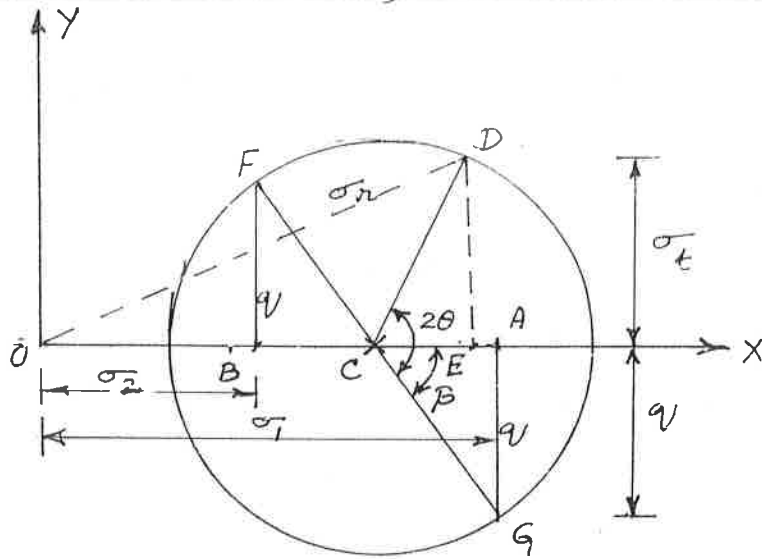
$$= \frac{\sigma_1 + \sigma_2}{2} \sin 2\theta = \sigma_t.$$
- Resultant stress σ_n ; $OD = \sqrt{OE^2 + ED^2}$

$$= \sqrt{\sigma_n^2 + \sigma_t^2}$$

$$= \sigma_n.$$

(16)

Mohr's Circle for Normal Stresses and Shear Stress 'q'



PROCEDURE

- On axis OX draw $OA = \sigma_1$ and $OB = \sigma_2$.
- Draw verticals at A and B such that $AG = FB = q$.
- Join FG to cut AB at C .
- With C as centre and CF and CG as radius draw a circle.
- Draw CD at an angle 2θ with $C.G$.
- Drop perpendicular DE on X -axis.
- OE gives normal stress σ_n , DE gives tangential stress σ_t and OD gives resultant stress σ_r .

PROOF

$$\begin{aligned}
 \rightarrow OE &= OC + CD \cos(2\theta - \beta) \\
 &= OC + CD \cos 2\theta \cos \beta + CD \sin 2\theta \sin \beta \\
 &= \frac{\sigma_1 + \sigma_2}{2} + (CD \cos \beta) \cos 2\theta + (CD \sin \beta) \sin 2\theta \\
 &= \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + q \sin \theta \\
 &= \sigma_n
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow DE &= CD \sin(2\theta - \beta) \\
 &= CD [\sin 2\theta \cos \beta - \cos 2\theta \sin \beta] \\
 &= (CD \cos \beta) \sin 2\theta - (CD \sin \beta) \cos 2\theta. \\
 &= \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - q \cos 2\theta. \\
 &= \sigma_t.
 \end{aligned}$$

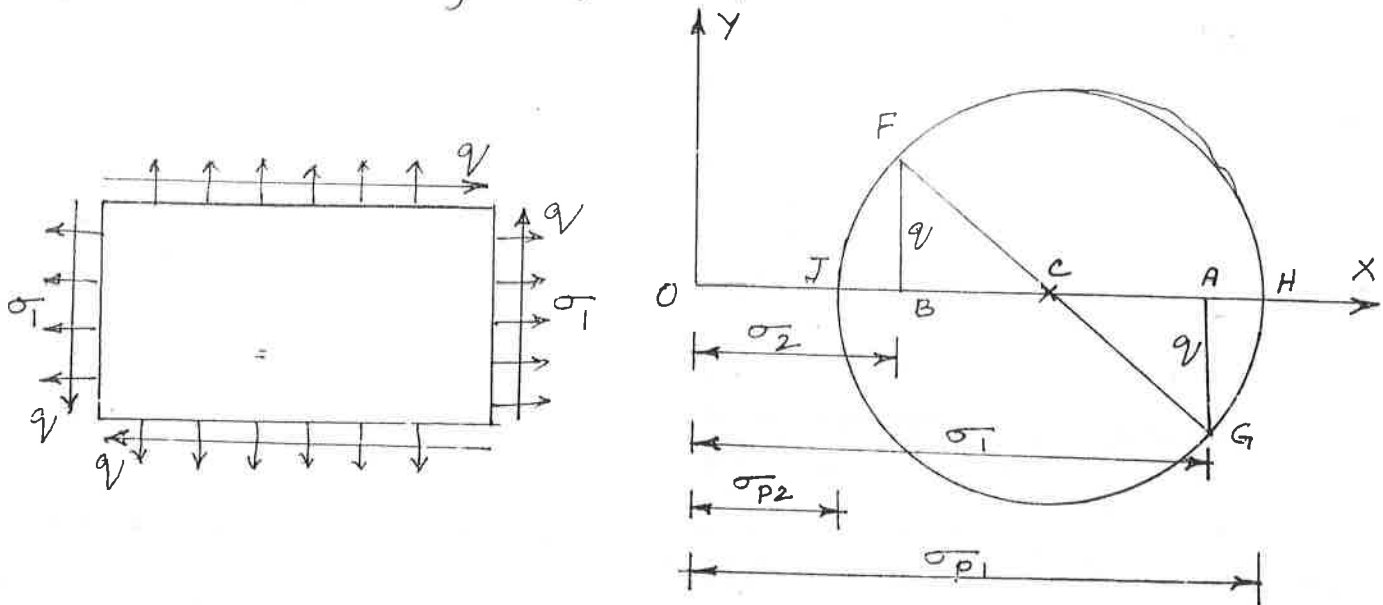
$$\begin{aligned}
 \rightarrow OD &= \sqrt{OE^2 + ED^2} \\
 &= \sqrt{\sigma_n^2 + \sigma_t^2} \\
 &= \sigma_n
 \end{aligned}$$

PRINCIPAL STRESSES AND PRINCIPAL PLANES:

- principal planes are the planes across which the stresses are wholly normal, i.e. no shear stress exist along these planes.
- The normal stresses across the principal planes are known as principal stresses.
- In any strained material there exist two principal planes mutually perpendicular to each other, they are:
 - (a) Major principal plane: The plane which carry the maximum normal stress.
 - (b) Minor principal plane: The plane which carry the minimum normal stress.

(18)

Mohr's Circle for PRINCIPAL STRESSES :



- On axis OX mark $OA = \sigma_1$; $OB = \sigma_2$
- At 'A' and 'B' erect perpendiculars AG and FB = q
- Join FG Cutting AB at C
- With C as centre and $CG = CF$ as radius draw a circle cutting OX at H and J.
- OH gives major principal stress σ_{p1} .
- OJ gives minor principal stress σ_{p2} .

PROOF

- $OH = OC + CH$
 $= OC + CG$
 $= OC + \sqrt{CA^2 + AG^2}$
 $= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$
 $= \sigma_{p1} \text{ (MAJOR PRINCIPAL STRESS)}$
- $OJ = OC - CJ$ ($CJ = CF = \sqrt{CB^2 + FB^2}$)
 $= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$
 $= \sigma_{p2} \text{ (MINOR PRINCIPAL STRESS)}$

THEORIES OF FAILURES.

Maximum Principal Stress Theory :

- It is simplest and oldest theory. It is also known as Rankin's Theory.
- It states that when the maximum principal stress σ_{P1} is equal to yield point stress, permanent deformation takes place.
- Or the minimum principal stress σ_{P2} (compressive stress) is equal yield point stress in simple compression.

Thus $\sigma_{P1} = \sigma_y$ (yield point stress in tension)

$\sigma_{P2} = \sigma_y'$ (yield point stress in compression)

- If maximum principal stress is the design criterion, it should not exceed the working stress σ .

$$\sigma_{P1} \leq \sigma$$

DRAWBACKS :

- This theory disregards the effect of other principal stresses and of the shearing stresses on the other planes through the element.
- The theory works satisfactorily for brittle material.
- The theory is not suitable for material which fails by yielding.

Maximum principal strain Theory:

- It is also known as Saint Venant's Theory.
- It states a ductile material begins to yield when the maximum principal strain reaches yield strain in simple tension.
- Or when the maximum principal strain (compressive) equals the yield point strain in simple compression.

In 3-D stress system the strains are

$$e_1 = \frac{\sigma_{p1}}{E} - \frac{1}{mE} (\sigma_{p2} + \sigma_{p3}) \quad \text{--- (1)}$$

$$e_2 = \frac{\sigma_{p2}}{E} - \frac{1}{mE} (\sigma_{p1} + \sigma_{p3}) \quad \text{--- (2)}$$

$$e_3 = \frac{\sigma_{p3}}{E} - \frac{1}{mE} (\sigma_{p2} + \sigma_{p1}) \quad \text{--- (3)}$$

The above equations may be written in terms of stress.

Max. principal strain $e_1 = e_y$ (yield point tensile strain)

$$\frac{\sigma_1}{E} = \frac{\sigma_y}{E} \Rightarrow \boxed{\sigma_1 = \sigma_y}$$

For 3-D stress. $\boxed{\sigma_1 - \frac{1}{m} (\sigma_2 + \sigma_3) = \sigma_y} \quad \text{--- (4)}$

For 2-D stress. $\boxed{\sigma_1 - \frac{\sigma_2}{m} = \sigma_y} \quad \text{--- (5)}$

or $\sigma_1 - \frac{\sigma_2}{m} \leq \sigma$ ($\sigma =$ working stress)

Drawbacks:

- The above theory is invalid for many cases for example when $\sigma_1 = \sigma_2$

From Eqn (5) $\sigma_1 - \frac{\sigma_2}{m} = \sigma_y$

$$\sigma_1 = \sigma_y \cdot \frac{m}{(m-1)}$$

$\sigma_1 > \sigma_y$ which is not supported by

Maximum Shear Stress Theory:

- This theory was suggested by J. J. Guest.
- The theory assumes that yielding begins when the maximum shear stress equals the yielding shearing stress developed in simple tension.
- Maximum Shear Stress $\tau_{\max} = \frac{\sigma_{P1} - \sigma_{P2}}{2}$

Maximum Shear Stress in Simple Tension $\tau_{\max} = \frac{\sigma_y}{2}$

Equating $\sigma_{P1} - \sigma_{P2} = \sigma_y$

- This theory is applicable to ductile material.

DRAWBACKS

- The theory does not give accurate results for the state of stress of pure shear.
- The theory is not applicable for triaxial tensile stress of nearly equal magnitude.

Maximum Strain Energy Theory:

- This theory is suggested by Beltrami, the theory is generally known as Haigh's Theory.
- It states that if a body is brought to a particular state by different methods then the work done by passing from the initial to final state will be independent of the method applied.
- When a material is caused to take permanent set by stress which increases from zero, then the initial strain energy is independent of nature of the stresses and is almost constant in value.
- The theory states that inelastic action at any point in a body due to any state of stress begins only when the energy per unit volume absorbed at the point is equal to the energy absorbed per unit volume of the material when subjected to the elastic limit under a uniaxial stress.
- In a 3-d stress system the strain energy per unit volume is given by
$$u = \frac{1}{2E} \left[\sigma_{p1}^2 + \sigma_{p2}^2 + \sigma_{p3}^2 - \frac{2\sigma_{p1}\sigma_{p2} + \sigma_{p2}\sigma_{p3} + \sigma_{p3}\sigma_{p1}}{m} \right]$$
 where σ_{p1} , σ_{p2} and σ_{p3} are of same sign, hence the yield criterion may be represented
i.e.
$$\sigma_y^2 = \sigma_{p1}^2 + \sigma_{p2}^2 + \sigma_{p3}^2 - \frac{2\sigma_{p1}\sigma_{p2} + 2\sigma_{p2}\sigma_{p3} + 2\sigma_{p3}\sigma_{p1}}{m}$$

→ For 2-D stress $\sigma_3 = 0$.

$$\Rightarrow \sigma_y^2 = \sigma_{p1}^2 + \sigma_{p2}^2 - \frac{2\sigma_{p1}\sigma_{p2}}{m}$$

If σ is working stress in the material the above equation becomes $\sigma_{p1}^2 + \sigma_{p2}^2 - \frac{2\sigma_{p1}\sigma_{p2}}{m} \leq \sigma^2$

MAXIMUM SHEAR STRAIN ENERGY THEORY:

→ This theory is also known as Energy of Distorsion Theory

→ It states that inelastic action at any point in a body under combination of stresses begins when the strain energy of distorsion absorbed at the point is equal to the strain energy of distorsion absorbed at any point in a bar stressed to the elastic limit under a state of uniaxial stress.

→ In 3-D stress system

$$\frac{1}{2}(\sigma_{p1} - \sigma_{p2})^2 + \frac{1}{2}(\sigma_{p2} - \sigma_{p3})^2 + \frac{1}{2}(\sigma_{p3} - \sigma_{p1})^2 = \sigma_y^2$$

→ In 2-D stress system $\sigma_3 = 0$

$$\Rightarrow \sigma_{p1}^2 + \sigma_{p2}^2 - \sigma_{p1}\sigma_{p2} = \sigma_y^2$$

→ If ' σ ' is working stress in the material

$$\Rightarrow \sigma_{p1}^2 + \sigma_{p2}^2 - \sigma_{p1}\sigma_{p2} \leq \sigma^2$$

High Lights

- ① If a bar is subjected axial force 'P', then normal stress σ_n , tangential stress σ_t and resultant stress σ_r on an inclined plane are given as

$$\rightarrow \sigma_n = \sigma_1 \cos^2 \theta$$

$$\rightarrow \sigma_t = \frac{\sigma_1}{2} \sin 2\theta$$

$$\rightarrow \sigma_n = \sigma_1 \cos \theta$$

where $\sigma_1 = \frac{P}{A}$ = axial stress.

$$\sigma_1 = \frac{P}{A} \quad \sigma_2 = \frac{P}{A}$$



- ② Member subjected to stress in x & y direction.

→ Normal stress σ_n

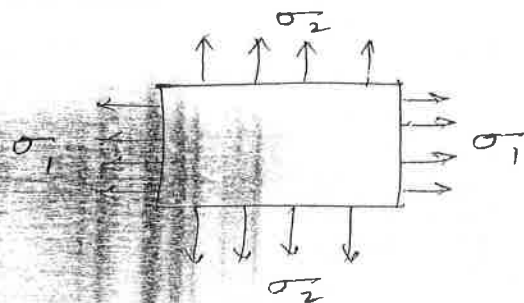
$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

→ Tangential stress σ_t

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

→ Resultant stress $\sigma_r = \sqrt{\sigma_1^2 \cos^2 \theta + \sigma_2^2 \sin^2 \theta}$

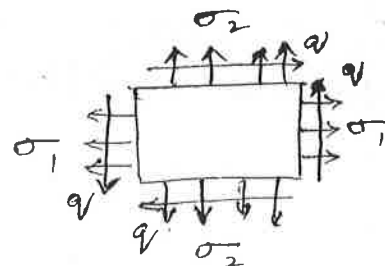
Angle of Slope of σ_r , $\tan \phi = \frac{\sigma_t}{\sigma_n}$



- ③ Member subjected axial stresses σ_1 and σ_2 along x and y-axis along with shear stress 'q'.

$$\rightarrow \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + q \sin 2\theta$$

$$\rightarrow \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - q \cos 2\theta$$



④ Major principal stress σ_{p1}

$$\sigma_{p1} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

→ Minor principal stress σ_{p2}

$$\sigma_{p2} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

→ Maximum shear stress q_{max} .

$$q_{max} = \frac{\sigma_{p1} - \sigma_{p2}}{2}$$

SHORT QUESTIONS:

Q1: Define uniaxial and biaxial stress system.

Ans1: If ~~the~~ a member is subjected to stress along one axis it is known as uniaxial stress.
If the member is subjected to stress along two axes (x and y), it is known as biaxial stress system.

Q2: Define principal planes.

Ans2: The planes on which the stresses are wholly normal and no shear stress act on it, is known as principal planes.

Q3: What are principal stresses?

Ans3: The stress which act on principal planes are principal stresses. σ_{p1} and σ_{p2} .

Q4: How many principal stresses are there what are they?

Ans4: There are two principal stresses, they are

(a) Major principal stress. σ_{p1}

(b) Minor principal stress. σ_{p2} .

→ Major principal ^{stress} is that which carry maximum normal stress.

→ Minor principal stress is that which carry minimum normal stress.

Q5: What are the magnitudes of Major principal stress and minor principal stress.

Ans 5: Major principal stress σ_{p1}

$$\sigma_{p1} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

$$\text{Minor principal stress } \sigma_{p2} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

Exercise Questions:

① At a point in a member tensile stresses are $\sigma_1 = 80 \text{ N/mm}^2$ and $\sigma_2 = 40 \text{ N/mm}^2$ mutually perpendicular to each other. Find.

(a) Normal stress σ_n

(b) Tangential stress σ_t

(c) Resultant stress σ_R on a plane at 30° with major principal plane.

Ans $\sigma_n =$

$\sigma_t =$

$\sigma_R =$

② At a point in a member a tensile stress $\sigma_1 = 80 \text{ N/mm}^2$ and compressive stress $\sigma_2 = 40 \text{ N/mm}^2$ act perpendicular to each other. Determine

(a) Normal stress σ_n .

(b) Tangential stress σ_t

(c) Resultant stress σ_R on a plane making 30° with major principal plane.

Ans $\sigma_n =$

$\sigma_t =$

$\sigma_R =$

③ Define principal planes and principal stresses.

④ What are Major principal stress and Minor principal stress? Write the expression to find their magnitude in terms of σ_1 , σ_2 and θ .

- ⑤ A rectangular block is subjected to a tensile stresses of $\sigma_1 = 150 \text{ N/mm}^2$ along x-axis and a tensile stress of $\sigma_2 = 75 \text{ N/mm}^2$ along y-axis, along with a shear stresses of 50 N/mm^2 on same planes

Determine

- ① Major principal stress σ_{p1}
 - ② Minor principal stress σ_{p2}
 - ③ Maximum Shear stress $\tau_{\max}$.
- ⑥ Name the different theories of failure. Explain any two of them.
- ⑦ What are the drawbacks of Maximum principal stress theory and principal strain theory.

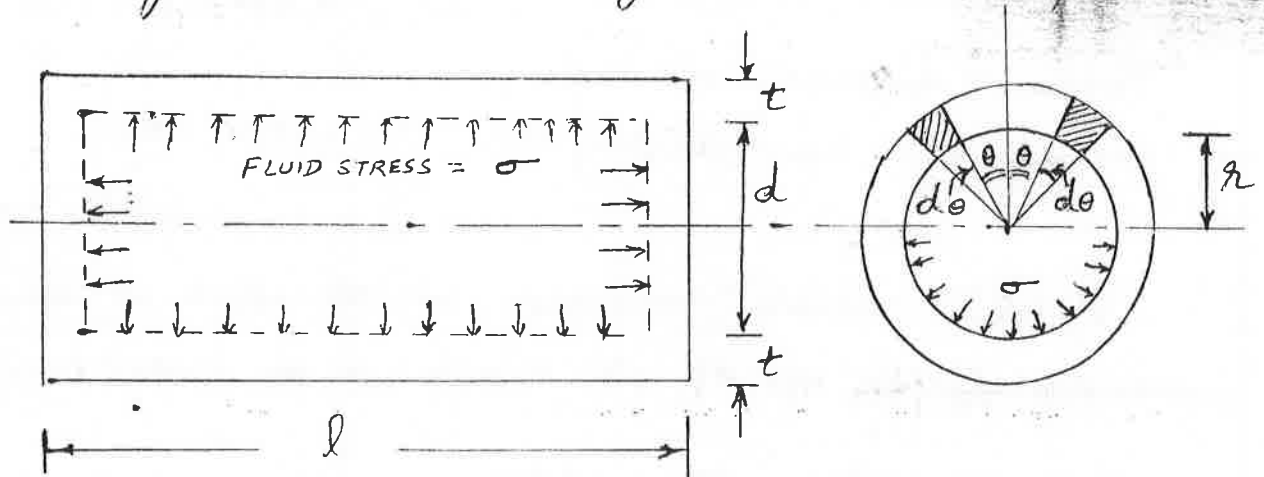
unit VII THIN CYLINDERS

Shell type structures are commonly used as tanks boilers etc. The walls of these structures are subjected to pressure from fluids or gases stored in them;

→ If the thickness of wall of the shell ' t ' is less than $\frac{1}{10}$ to $\frac{1}{15}$ of the diameter of the shell it is considered as thin shell.

→ The stresses developed in the thickness of walls are assumed to be uniformly distributed.

Thin Cylindrical Shell Subjected to Internal pressure:



Consider a thin cylinder shown in figure above, whose thickness $t < \frac{1}{10}$ to $\frac{1}{15}$ of dia ' d '. The cylinder is filled with fluid exerting a stress ' σ ' on the walls of cylinder.

consider two elementary strips of the shell subtending angle $\delta\theta$ at centre and making angle θ with vertical.
 σ = fluid stress acting on the walls of the cylinder.

It is required to find

(a) Hoop stress σ_H (also known as circumferential stress) developed in the walls of the cylinder due to fluid pressure σ .

(b) Longitudinal stress " σ_L " developed in the walls of cylinder due to fluid pressure " σ ".

→ To find Hoop stress σ_H .

consider the elementary strips of the shell

Normal force on one
elementary strip }

$$\begin{aligned} dP_m &= \text{stress} \times \text{area} \\ &= \text{stress} \times \text{Length of strip} \times \text{Length of cylinder} \\ &= \sigma \times r d\theta \times l \end{aligned}$$

Vertical component of
force due to two strips }

$$\begin{aligned} dP &= 2 \times dP_m \times \cos\theta \\ &= 2 \times \sigma \times r d\theta \times l \times \cos\theta \quad \text{--- (1)} \end{aligned}$$

The total radial pressure on the upper or lower portion of the shell is known as the BURSTING FORCE

and is given by

$$\begin{aligned} P &= \int dP \\ &= \int_0^{\pi/2} 2 \cdot \sigma \cdot r \cdot l \cdot \cos\theta \cdot d\theta \\ &= 2 \cdot \sigma \cdot r \cdot l \int_0^{\pi/2} \cos\theta d\theta \\ &= 2 \sigma r l \left[\sin\theta \right]_0^{\pi/2} \\ &= \sigma d l \end{aligned}$$

(2)

Let σ_H = Hoop stress in the walls of cylinder.

$$\begin{aligned} \text{Resisting force} &= \sigma_H (2t \times l) \quad \text{--- (2)} \\ &= \text{stress} \times \text{two elements of thickness 't'} \times \\ &\quad \text{Length of cylinder.} \end{aligned}$$

Equating (1) and (2)

$$\sigma_H \times 2t \times l = \sigma \times d \times l.$$

$$\text{Hoop STRESS} \quad \boxed{\sigma_H = \frac{\sigma \cdot d}{2t}} \quad \text{--- (3)}$$

Longitudinal Stress on the End Caps of Cylinder:

$$\left. \begin{array}{l} \text{Bursting force } P_B \text{ due to} \\ \text{fluid pressure on the} \\ \text{end caps of cylinder} \end{array} \right\} \begin{aligned} P_B &= \text{Stress} \times \text{Area of end cap.} \\ &= \sigma \times \frac{\pi}{4} d^2 \quad \text{--- (4)} \end{aligned}$$

→ Bursting force is resisted by joint on the circumference of the cap.

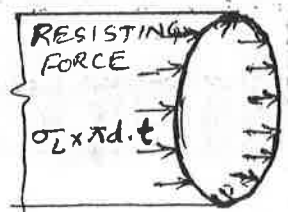
If σ_L = Longitudinal stress, then resisting

$$\text{force} = \sigma_L \times \pi d \times t \quad \text{--- (5)}$$

$$\text{Equating (4) and (5)} \quad \sigma_L \cdot \pi d \cdot t = \sigma \times \frac{\pi}{4} d^2$$

LONGITUDINAL STRESS

$$\boxed{\sigma_L = \frac{\sigma d}{4t}} \quad \text{--- (6)}$$



(4)

If η is the efficiency of the joint then

$$\text{Hoop Stress } \sigma_H = \frac{\sigma \cdot d}{2t \cdot \eta} \quad \text{and}$$

$$\text{Longitudinal Stress } \sigma_L = \frac{\sigma \cdot d}{4t \cdot \eta}$$

CIRCUMFERENTIAL STRAIN e_c

$$\text{Strain} = \frac{\text{Stress}}{E} - \frac{1}{m} \frac{\text{Longitudinal Stress}}{E}$$

$$e_c = \frac{\sigma_H}{E} - \frac{1}{m} \frac{\sigma_L}{E}$$

$$= \frac{\sigma \cdot d}{2tE} - \frac{1}{m} \frac{\sigma \cdot d}{4tE}$$

$$e_c = \frac{\sigma \cdot d}{2tE} \left(1 - \frac{1}{2m}\right)$$

LONGITUDINAL STRAIN e_L

$$e_L = \frac{\sigma_L}{E} - \frac{\sigma_H}{mE}$$

$$= \frac{\sigma d}{4tE} - \frac{\sigma d}{m \cdot 2tE}$$

$$e_L = \frac{\sigma d}{2tE} \left(\frac{1}{2} - \frac{1}{m}\right)$$

VOLUMETRIC STRAIN e_v

$$e_v = \frac{\text{Change volume } \delta V}{\text{Original Volume } V}$$

$$V = \frac{\pi d^2}{4} \times l \Rightarrow \text{Differentiating partially}$$

$$\delta V = \frac{\pi d^2}{4} \delta l + \frac{\pi \cdot l}{4} \cdot 2d \cdot \delta d$$

$$\Rightarrow e_v = \frac{\delta V}{V} = \frac{\delta l}{l} + 2 \cdot \frac{\delta d}{d} = e_L + 2e_c$$

$$= \frac{\sigma d}{4tE} - \frac{\sigma d}{m \cdot 2tE} + 2 \left[\frac{\sigma \cdot d}{2tE} - \frac{\sigma d}{m \cdot 4tE} \right]$$

Equating resisting force to bursting force

$$\sigma_1 \cdot \pi d t = \sigma \cdot \frac{\pi d^2}{4}$$

RESISTING STRESS $\sigma_1 = \frac{\sigma d}{4t}$

$$\sigma_1 = \frac{\sigma \cdot d}{4t \cdot \eta}$$

If ' η ' is the efficiency of joint

→ STRAIN in any direction ' e ' = $\frac{\delta d}{d}$.

$$e = \frac{\sigma_1}{E} - \frac{\sigma_1}{mE}$$

$$= \frac{\sigma_1}{E} \left(1 - \frac{1}{m}\right)$$

→ Increase in DIAMETER δd :

Increase in dia δd may be obtained from the expression $\frac{\delta d}{d} = e$

→ VOLUMETRIC STRAIN $e_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$

Original volume $V = \frac{4}{3} \pi r^3$
 $= \frac{4}{3} \pi \left(\frac{d}{2}\right)^3$
 $= \frac{\pi d^3}{6}$

Differentiating $\delta V = \frac{3\pi d^2}{6} \delta d$
 $= \frac{\pi d^2}{2} \delta d$

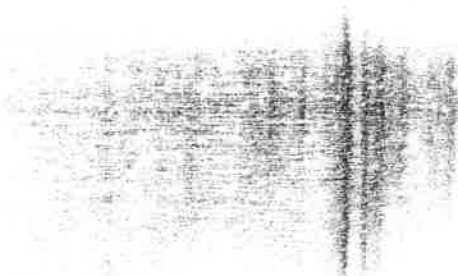
Volumetric strain $e_v = \frac{\frac{\pi d^2}{2} \delta d}{\frac{\pi d^3}{6}} = \frac{\pi d^2}{2} \delta d \times \frac{6}{\pi d^3}$

$$e_v = 3 \frac{\delta d}{d}$$

$$e_v = 3e$$

1

2



Prob 9: JNTU Suppl. May/June 2009, Suppl Set (4) Code 43001

- (b) A spherical vessel 1.5 m diameter is subjected to an internal pressure of 2 N/mm^2 . Find the thickness of the plate required if maximum stress is not to exceed 150 N/mm^2 and joint efficiency is 75%.

SOLUTION:

$$d = 1.5 \text{ m} = 1.5 \times 10^3 \text{ mm} ; \sigma = 2 \text{ N/mm}^2$$

$$\sigma_1 = 150 \text{ N/mm}^2 ; \eta = 75\% = \frac{75}{100} = \underline{0.75}$$

$$\text{Resisting Stress } \sigma_1 = \frac{\sigma \cdot d}{4 t \cdot \eta}$$

$$150 = \frac{2 \times 1.5 \times 10^3}{4 t \times 0.75}$$

$$\boxed{t = 6.67 \text{ mm.}}$$

Prob 10: JNTU May/June Suppl. 2009, NR. Code NO NR 210102

A cylindrical air receiver for a compressor is 2 m in internal diameter and made of plate 15 mm thick. If the hoop stress is not to exceed 90 N/mm^2 and longitudinal stress not to exceed 60 N/mm^2 , find the maximum safe air pressure.

SOLUTION:

$$\rightarrow \text{Hoop Stress } \sigma_H = \frac{\sigma \cdot d}{2 t} \Rightarrow 90 = \frac{\sigma \times 2000}{2 \times 15}$$

$$\text{air pressure } \sigma = 1.35 \text{ N/mm}^2$$

$$\begin{aligned} \rightarrow \text{Longitudinal Stress } \sigma_L &= \frac{\sigma \cdot d}{4 t} \Rightarrow 60 = \frac{\sigma \times 2000}{4 \times 15} \\ &= \underline{1.8 \text{ N/mm}^2} \end{aligned}$$

Safe air pressure is lesser of two = 1.35 N/mm}^2

Prob 11: JNTU Nov 2007 Main Set No. ① Code NO: R050210101.

(a) A water main 600 mm dia contains water at a pressure head of 100 m. Find the thickness of the metal required if the permissible stress is 30 N/mm^2 .

(b) A vessel in the shape of a spherical shell 800 mm in diameter, 10 mm shell thickness is completely filled with a fluid at atmospheric pressure. Additional fluid is then pumped in, till the pressure increases by 5 N/mm^2 . Find the volume of additional fluid.

Take $E = 2 \times 10^5 \text{ N/mm}^2$, $\mu = \frac{1}{m} = 0.25$.

SOLUTION:

(a) pressure due to water head $P_w = \frac{\pi \times 0.6^2}{4} \times 100 \times 1000 \text{ Kg}$
(Density of water = 1000 Kg/m^3)
 $= 28,274.33 \text{ Kg}$
 $= 282743.3 \text{ N}$

Stress on the walls of main $\sigma = \frac{P_w}{(\pi d) t}$
 $= \frac{282743.3}{(\pi \times 600) t}$

$$\sigma = \frac{150}{t}$$

permissible Hoop Stress $\sigma_H = \frac{\sigma \cdot d}{2 t}$

$$30 = \frac{150}{t} \times \frac{600}{2 t}$$

$$t = 38.72 \text{ mm} \checkmark$$

(b) $d = 800 \text{ mm}$; $t = 10 \text{ mm}$; $\sigma = 5 \text{ N/mm}^2$.

Resisting stress in spherical shell.

$$\sigma_1 = \frac{\sigma d}{4t}$$

$$= \frac{5 \times 800}{4 \times 10}$$

$$= \underline{100 \text{ N/mm}^2}$$

Radial strain $e = \frac{\sigma_1}{E} - \frac{1}{m} \frac{\sigma_1}{E}$

$$= \frac{100}{2 \times 10^5} - \frac{0.25 \times 100}{2 \times 10^5}$$

$$= \underline{3.75 \times 10^{-4}}$$

Volumetric strain $e_v = 3.e$

$$e_v = \frac{\delta V}{V}$$

$$= 3 \times 3.75 \times 10^{-4}$$

$$= \underline{1.125 \times 10^{-3}}$$

$$\frac{\delta V}{V} = 1.125 \times 10^{-3}$$

$$\delta V = 1.125 \times 10^{-3} \times \frac{4}{3} \pi \times 400^3$$

$$= \underline{301592.89 \text{ mm}^3}$$

→ Volume of additional fluid = 301592.89 mm³

SHORT QUESTIONS :

Q1: A cylinder is considered as a thin cylinder if

(a) Thickness $< \frac{\text{dia}}{2}$ (b) Thickness $t < \frac{d}{3}$.

(c) Thickness t is between $\frac{d}{10}$ to $\frac{d}{15}$ (d) None.

Q2: If a thin cylinder is subjected to a fluid pressure σ , and if 'r' is the radius of cylinder the Hoop stress σ_H is given by the expression

(a) $\sigma_H = \frac{\sigma \cdot d}{4t}$ (b) $\sigma_H = \frac{\sigma \cdot d}{3t}$ (c) $\sigma_H = \frac{\sigma \cdot d}{2t}$ (d) None.

Q3: Longitudinal stress σ_L in a thin cylinder of radius 'r' and internal fluid pressure ' σ ' is given as

(a) $\sigma_L = \frac{\sigma \cdot d}{4t}$ (b) $\sigma_L = \frac{\sigma \cdot d}{3t}$ (c) $\sigma_L = \frac{\sigma \cdot d}{2t}$ (d) None.

Q4: Maximum shear stress $\tau_{\max}$ in a thin cylinder is given by the expression

(a) $\tau_{\max} = \frac{\sigma \cdot d}{2t}$ (b) $\tau_{\max} = \frac{\sigma \cdot d}{6t}$ (c) $\tau_{\max} = \frac{\sigma \cdot d}{4t}$ (d) None.

Q5: If ' η ' is the efficiency of joint of a thin cylinder of dia 'd' and thickness 't' subjected to internal stress ' σ ' then the circumferential stress is given by the expression $\sigma_H = \dots$ and longitudinal stress σ_L is given by the expression $\dots$

Q⑥ A thin spherical shell of thickness 't' and diameter 'd' is subjected to internal stress ' σ ' the resisting stress σ_1 is given by the expression

(a) $\sigma_1 = \frac{\sigma \cdot d}{2}$ (b) $\sigma_1 = \frac{\sigma d}{2t}$ (c) $\sigma_1 = \frac{\sigma \cdot d}{4}$ (d) $\frac{\sigma \cdot d}{4 \cdot t}$

Q⑦ A thin spherical shell subjected to resisting stress σ_1 . strain in radial direction is obtain as -----

(a) $e_d = \frac{\delta d}{d}$ (b) $e_d = \frac{\sigma_1}{E} - \frac{\sigma_1}{mE}$ (c) $\frac{\sigma_1}{E} (1 - \frac{1}{m})$ (d) All the three.

Q⑧ Volumetric strain e_v in a thin spherical shell subjected to internal stress

(a) $e_v = 3e$ (b) $e_v = 3 \cdot \frac{\delta d}{d}$ (c) $3 \frac{\sigma_1}{E} (1 - \frac{1}{m})$ (d) All the three.

Q⑨ If ' η ' is the efficiency of joint of a thin spherical shell subjected to internal stress ' σ '. Resisting stress σ_1 is given by the expression

(a) $\sigma_1 = \eta \frac{\sigma \cdot d}{2}$ (b) $\sigma_1 = \frac{\sigma \cdot d}{2\eta}$ (c) $\sigma_1 = \eta \cdot \frac{\sigma \cdot d}{4t}$ (d) $\sigma_1 = \frac{\sigma \cdot d}{4t \cdot \eta}$

Q⑩ A thin spherical shell of 2m internal diameter experiences a radial strain $e = 1 \times 10^{-3}$. Find the change in diameter.

Ans⑩ Radial strain $e = \frac{\delta d}{d} \Rightarrow 1 \times 10^{-3} = \frac{\delta d}{2000}$

change in diameter $\delta d = 2 \text{ mm}$

Exercise Questions.

- ① A thin cylinder of diameter 'd' and thickness 't' is subjected to internal pressure $\sigma \text{ N/mm}^2$. Derive expressions to find (a) Circumferential stress or Hoop stress σ_H and (b) Longitudinal stress.
- ② Obtain the expressions for finding radial strain and Longitudinal strain. Use these expressions to find change in dia δd and change in volume δV .
- ③ A thin spherical shell of diameter 'd' and thickness 't' is subjected to internal pressure (Stress) ' σ '. Derive the expression for resisting stress developed in the shell.
- ④ A thin seamless pipe of 0.8 m diameter is subjected to internal fluid pressure of 2 N/mm^2 . Permissible tensile stress may be taken as 80 N/mm^2 . Find the thickness of pipe.
(Ans $t = 10 \text{ mm}$)
- ⑤ A thin cold drawn ^{steel} pipe of 60 mm internal diameter and 2 mm wall thickness. Calculate the bursting stress if permissible stress in steel is 240 N/mm^2 .
(Ans 16 N/mm^2)

(23)

⑥ A thin cylindrical pipe of 0.35 m is subjected to an internal pressure σ which develops a radial strain of 4×10^{-4} . If thickness of shell is 5 mm , find

- (a) Internal pressure σ .
- (b) Hoop stress σ_H
- (c) Longitudinal stress σ_L .

Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\frac{1}{m} = \mu = 0.3$.

Ans $\sigma = 2.69 \text{ N/mm}^2$

$\sigma_H = 94.15 \text{ N/mm}^2$

$\sigma_L = 47.07 \text{ N/mm}^2$

and 2 m long

⑦ A cylinder 1.2 m in diameter, is subjected to an internal pressure of 3 N/mm^2 . Thickness of the cylinder is 10 mm , and $E = 2 \times 10^5 \text{ N/mm}^2$ and $\frac{1}{m} = 0.3$. Determine

- (a) Circumferential or Hoop stress σ_H
- (b) Longitudinal stress σ_L .
- (c) Circumferential strain e_c
- (d) Longitudinal strain e_L
- (e) Change in volume δV .

Ans:

$\sigma_H = 180 \text{ N/mm}^2$

$\sigma_L = 90 \text{ N/mm}^2$

$e_c = 7.65 \times 10^{-4}$

$e_L = 1.80 \times 10^{-4}$

$\delta V = 3.86 \times 10^6 \text{ mm}^3$

⑧ A spherical shape vessel 1000 mm in dia is filled with a liquid at a pressure of 1.5 N/mm^2 , thickness of shell is 8 mm . Determine (a) Increase in diameter of shell (b) Increase in volume of the shell

Take $E = 2 \times 10^5 \text{ N/mm}^2$

$\frac{1}{m} = 0.3$.

Ans

$\delta d = 0.164 \text{ mm}$

$\delta V = 257.7 \times 10^6 \text{ mm}^3$

Unit VIII THICK CYLINDERS.

In thick cylinders circumferential stress σ_H does not remain constant along the thickness of the cylinder but varies along the section.

Radial pressure σ_r is no longer negligible since a thick cylinder has to resist heavy internal pressure.

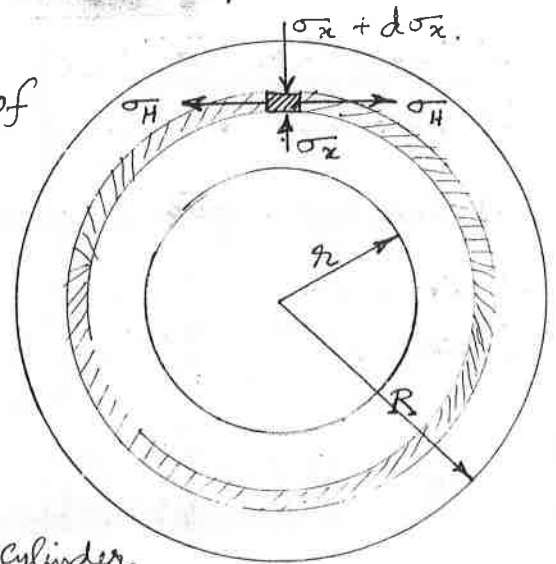
LAME'S THEORY (TO FIND HOOP STRESS σ_H and Radial Stress σ_r)

The problem of thick cylinders was solved by Lamé based on the following assumptions:

- ① The material of cylinder is homogeneous and isotropic.
- ② Plane sections of the cylinder perpendicular to the longitudinal axis remain plane under pressure.

Consider a thick cylinder of internal radius 'r' and external radius 'R'.

Consider an elemental ring internal radius 'x' and thickness dx .



Let σ_H = Hoop stress in the cylinder

σ_L = Longitudinal stress

σ_r = Radial stress at distance 'x' from centre

Based on assumption ②: Longitudinal strain e_L is constant.

$$e_L = \frac{\sigma_L}{E} - \frac{\sigma_H}{mE} + \frac{\sigma_r}{mE} \quad \text{--- ①}$$

→ Consider equilibrium of half of ring.

$$\begin{aligned} \rightarrow \text{Bursting force} &= (\sigma_x \cdot 2xl) - (\sigma_x + d\sigma_x) 2(x+dx)l \\ &= 2l(-\sigma_x \cdot dx - x \cdot d\sigma_x - dx \cdot d\sigma_x) \end{aligned}$$

Neglecting product of small quantities i.e. $dx \cdot d\sigma_x$,

$$\text{Bursting force} = -2l(\sigma_x \cdot dx + x \cdot d\sigma_x) \quad \text{--- (2)}$$

$$\rightarrow \text{Resisting force} = 2 \cdot \sigma_H l \cdot dx \quad \text{--- (3)}$$

Equating Resisting force and Bursting force

$$2 \cdot \sigma_H l \cdot dx = -2l(\sigma_x \cdot dx + x \cdot d\sigma_x)$$

$$\sigma_H = -\sigma_x - x \frac{d\sigma_x}{dx}$$

$$\text{or } \sigma_H + \sigma_x + x \cdot \frac{d\sigma_x}{dx} = 0 \quad \text{--- (4)}$$

$$\rightarrow \text{From Eqn (1)} \quad \frac{\sigma_L}{E} - \frac{\sigma_H}{mE} + \frac{\sigma_x}{mE} = \text{Constant}$$

Since σ_L , m and E are constant

$$\Rightarrow \sigma_H - \sigma_x = 2A \text{ (Constant)}$$

$$\sigma_H = \sigma_x + 2A \text{ Substituting in Eqn (4)}$$

$$(\sigma_x + 2A) + \sigma_x + x \frac{d\sigma_x}{dx} = 0$$

$$\frac{d\sigma_x}{dx} = - \frac{2(\sigma_x + A)}{x}$$

$$\text{or } \frac{d\sigma_x}{(\sigma_x + A)} = - \frac{2 dx}{x} \left[\int \frac{1}{x} = \log_e x \right]$$

$$\text{Integrating } \log_e (\sigma_x + A) = - \left\{ \log_e x^2 + \log_e B \right\}$$

where $\log B$ is a constant of integration.

Rearranging $\log_e (\sigma_x + A) = \log_e \frac{B}{x^2}$

Removing log $\sigma_x + A = \frac{B}{x^2}$

or $\boxed{\sigma_x = \frac{B}{x^2} - A}$ Radial pressure.

Substituting $\sigma_H - \sigma_x = 2A$ in the above expression.

$\sigma_H + 2A = \frac{B}{x^2} - A \rightarrow \sigma_H - 2A = \sigma_x$

$\boxed{\sigma_H = \frac{B}{x^2} + A}$ Hoop pressure.

The constants 'A' and 'B' can be obtained by applying end conditions (ie. Boundary conditions).

NOTE:

Radial pressure σ_x = pressure applied by material inside the cylinder at $x = r$ (internal radius)

$\sigma_x = 0$ at $x = R$ (External surface of cylinder)

$\frac{d}{dx} \left(\frac{2}{x} \right) = \log \frac{2}{x^2}$
 $\frac{d}{dx} \log \frac{2}{x^2} = \frac{2}{x^2}$
 $\frac{d}{dx} \log \frac{2}{x^2} = \frac{2}{x^2}$
 $\int \frac{2}{x^2} dx = \log \frac{2}{x^2}$

Subtracting (iii) from Equation (ii) we get

$$8 = \frac{B}{40000} - \frac{B}{90000} \Rightarrow \frac{9b - 4b}{360000} \Rightarrow \frac{5b}{360000}$$

$$\Rightarrow 8 = \frac{5b}{360000} \Rightarrow B = \frac{360000 \times 8}{5} \Rightarrow B = 576000$$

Substituting the value (iii) we get

$$0 = \frac{576000}{90000} - A \Rightarrow A = \frac{576000}{90000} = 6.4$$

Prob 1: JNTU May/June 2009 suppl. Set No ② Code 43001.

A cast iron pipe of 400 mm internal diameter and 100 mm thickness carries water under a pressure of 8 N/mm^2 . Determine the maximum and minimum intensities of Hoop stress across the section. Also sketch the radial pressure distribution and hoop stress distribution across the section.

SOLUTION: $d = 400 \text{ mm}$; $t = 100 \text{ mm}$; $\sigma = 8 \text{ N/mm}^2$
 $r = 200 \text{ mm}$; $D = 600 \text{ mm} \Rightarrow R = 300 \text{ mm}$

Radial pressure $\sigma_r = 8 \text{ N/mm}^2$ at radius $r = x = 200 \text{ mm}$.

$$\sigma_r = 0 \quad \text{at radius } R = x = 300 \text{ mm}$$

We know RADIAL PRESSURE $\sigma_r = \frac{B}{x^2} - A$ ——— ①

Substitute $x = 200 \text{ mm}$; $\sigma_r = 8 \text{ N/mm}^2$ } $\Rightarrow 8 = \frac{B}{200^2} - A$ ——— ②
in equation ①.

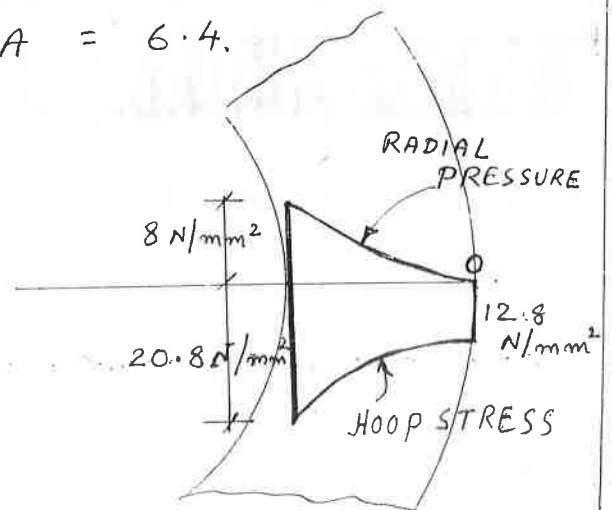
Again when $x = 300 \text{ mm}$.
Radial stress $\sigma_r = 0$ at SURFACE } $\Rightarrow 0 = \frac{B}{300^2} - A$ ——— ③
Substituting in ①.

Solving Equation ② and ③ $B = 576000$
 $A = 6.4$

HOOP STRESS $\sigma_H = \frac{B}{x^2} + A$

At $x = 200 \text{ mm}$. $\sigma_H = \frac{576000}{200^2} + 6.4$
 $= 20.8 \text{ N/mm}^2$

At $x = 300 \text{ mm}$ $\sigma_H = \frac{576000}{300^2} + 6.4$
 $= 12.8 \text{ N/mm}^2$



Prob 2: JNTU NOV 2007 - Regular Set No (3) Code R050210101.

Compare the values of max. and minimum hoop stresses for a cast steel cylindrical shell of 600mm external dia. and 400mm internal dia. subjected to a pressure of 30 N/mm^2 applied

(a) Internally.

(b) Externally.

SOLUTION: $D = 600 \text{ mm} \Rightarrow R = 300 \text{ mm}$,
 $d = 400 \text{ mm} \Rightarrow r = 200 \text{ mm}$.

Thickness $t = \frac{600 - 400}{2} = 100 \text{ mm}$

@ pressure applied internally

$$\text{Radial pressure } \sigma_x = \frac{B}{x^2} - A \quad \text{--- (1)}$$

at $x = 200 \text{ mm}$; $\sigma_x = 30 \text{ N/mm}^2$, Substituting in (1)

$$30 = \frac{B}{200^2} - A \quad \text{--- (2)}$$

at $x = 300 \text{ mm}$; $\sigma_x = 0$

$$0 = \frac{B}{300^2} - A \quad \text{--- (3)}$$

Subtracting (3) from (2) and simplifying $B = 2160000$.
and $A = 24$

HOOP STRESS:

$$\sigma_H = \frac{B}{x^2} + A$$

$$\begin{aligned} \text{At internal radius } x = 200 \text{ mm} \Rightarrow \sigma_H &= \frac{2160000}{200^2} + 24 \\ &= 78 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{At external radius } x = 300 \text{ mm} \Rightarrow \sigma_H &= \frac{2160000}{300^2} + 24 \\ &= 48 \text{ N/mm}^2 \end{aligned}$$

⑥ pressure applied Externally:

$$\text{Radial pressure } \sigma_x = \frac{B}{x^2} - A \quad \text{--- (1)}$$

at $x = 300 \text{ mm}$; $\sigma_x = 30 \text{ N/mm}^2$ Substituting in (1).

$$30 = \frac{B}{300^2} - A \quad \text{--- (2)}$$

at $x = 200 \text{ mm}$; $\sigma_x = 0$ Subst. in (1).

$$0 = \frac{B}{200^2} - A \quad \text{--- (3)}$$

Subtracting (3) from Eqn (2) and simplifying

$$B = -2160000.$$

$$A = -54.$$

HOOP STRESS σ_H

$$\sigma_H = \frac{B}{x^2} + A$$

ON SURFACE i.e

$$\begin{aligned} \text{at } x = 300 \text{ mm} ; \sigma_H &= \frac{-2160000}{300^2} - 54 \\ &= \underline{\underline{-78 \text{ N/mm}^2}} \end{aligned}$$

at inner surface

$$\begin{aligned} \text{i.e at } x = 200 \text{ mm} ; \sigma_H &= \frac{-2160000}{200^2} - 54 \\ &= \underline{\underline{-108 \text{ N/mm}^2}}. \end{aligned}$$



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COMPOUND CYLINDERS (SHRINKING ON)

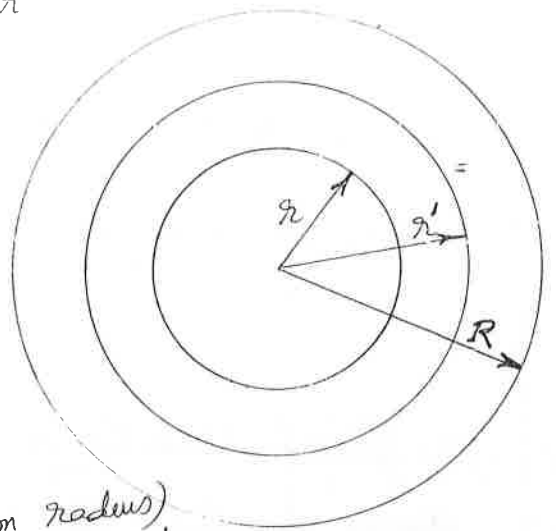
When one cylinder is shrunk on the other such an assembly is known as Compound cylinder.

Consider the compound cylinder shown in figure

Let r = Inner radius of internal tube.

r' = Outer radius of internal tube and inner radius of outer tube (common radius).

R = External radius of outer tube



LAME'S EQUATIONS FOR INNER TUBE

→ Radial pressure $\sigma_r = \frac{B}{r^2} - A$.

→ Circumferential Stress or Hoop Stress: $\sigma_H = \frac{B}{r^2} + A$.

LAME'S EQUATIONS FOR OUTER TUBE

→ Radial pressure $\sigma_r = \frac{B'}{r^2} - A'$.

→ Circumferential or Hoop Stress: $\sigma_H = \frac{B'}{r^2} + A'$.

In the above four equations the four unknowns A , B and A' , B' may be obtained by applying initial boundary conditions (Before passing the fluid).

Boundary Conditions (Initial conditions - Before admitting the fluid)

→ For Inner tube

Radial pressure $\sigma_r = 0$ at $r = R$.

Radial pressure $\sigma_r = \sigma'$ at $r = r'$ ($\sigma' =$ shrinking stress)

→ For outer tube

Radial pressure $\sigma_r = \sigma'$ at $r = r'$

$\sigma_r = 0$ at $r = R$.

Final conditions (After admitting the fluid).

Lame's Equations for COMPOUND CYLINDER under pressure

Radial pressure $\sigma_r = \frac{B''}{r^2} - A''$

Hoop stress $\sigma_H = \frac{B''}{r^2} + A''$

Boundary conditions

Radial pressure $\sigma_r = \sigma$ (fluid pressure) at $r = R$.

$\sigma_r = 0$ at $r = R$.

→ The final stresses in compound cylinder will be the algebraic sum of the initial stress and final stress due to fluid pressure.

Difference Of Radii For Shrinkage:

It is required to determine the necessary difference of radii at the junction to create the required shrinkage pressure.

Let r' = common radii of two tubes at junction.

δr_1 = Difference between outer radius of inner of inner tube and common radius r' .

δr_2 = Difference between inner radius of Outer Tube and common radius r' .

$$\begin{aligned}\delta r' &= \text{Difference in common radii before shrinking on.} \\ &= \delta r_1 + \delta r_2\end{aligned}$$

→ For the inner tube, the circumferential strain at the common radius r' is given by

$$\frac{\delta r_1}{r'} = \frac{1}{E} \left[\left(\frac{B}{r'^2} + A \right) + \frac{\sigma'}{m} \right] \quad \text{where } \sigma' = \text{shrinking stress.}$$

→ Similarly for the outer tube, the circumferential strain at the common radii r' is given by

$$\frac{\delta r_2}{r'} = \frac{1}{E} \left[\left(\frac{B'}{r'^2} + A' \right) + \frac{\sigma'}{m} \right]$$

From the above two equations

$$\frac{\delta r_1 + \delta r_2}{2} = \frac{\delta r'}{r} = \frac{1}{E} \left[\left(\frac{B'}{r'^2} + A' \right) - \left(\frac{B}{r'^2} + A \right) \right]$$

prob 3:

A compound cylinder formed by shrinking one tube onto another is subjected to an internal pressure of 65 N/mm^2 . At the initial condition (before fluid is admitted) the internal and external dia. of compound cylinder are 120 mm and 200 mm and the dia. at the junction is 150 mm . After shrinking the radial pressure at the common surface is 8 N/mm^2 . Determine the final stresses.

SOLUTION

$$\sigma_r = 65 \text{ N/mm}^2 \quad ; \quad r = 60 \text{ mm} \quad ; \quad R = 100 \text{ mm}.$$

$$\sigma' = 8 \text{ N/mm}^2 \quad ; \quad r' = 75 \text{ mm} \quad ; \quad \sigma_{R=100} = 0.$$

Lame's Equations for inner tube:

$$\rightarrow \text{Radial pressure } \sigma_r = \frac{B}{x^2} - A \quad \text{--- (1)}$$

$$\rightarrow \text{Hoop stress } \sigma_H = \frac{B}{x^2} + A \quad \text{--- (2)}$$

Lame's Equations for outer tube:

$$\rightarrow \text{Radial press. } \sigma_r = \frac{B'}{x^2} - A' \quad \text{--- (3)}$$

$$\rightarrow \text{Hoop stress } \sigma_H = \frac{B'}{x^2} + A' \quad \text{--- (4)}$$

Initial conditions: (Inner tube)

$$\text{At } x = 60 \quad ; \quad \sigma_r = 0 \quad \text{Substitute in (1)} \quad 0 = \frac{B}{60^2} - A \quad \text{--- (5)}$$

$$\text{At } x = 75 \quad ; \quad \sigma_r = 8 \text{ N/mm}^2 \quad \text{Subst (1)} \quad 8 = \frac{B}{75^2} - A \quad \text{--- (6)}$$

Subtracting (6) from (5) and solving.

$$B = -80,000$$

$$A = -22.22$$

$$\left. \begin{array}{l} \text{Hoop stress at } x = 60 \text{ mm} \\ \text{Subst A \& B in Eqn (2)} \end{array} \right\} \sigma_H = \frac{-80,000}{60^2} - 22.22$$
$$= \underline{-44.44 \text{ N/mm}^2} \text{ (Compressive)}$$

$$\text{Hoop stress at } x = 75 \text{ mm}; \sigma_H = \frac{-80,000}{75^2} - 22.22$$
$$= \underline{-36.44 \text{ N/mm}^2} \text{ (Compressive)}$$

Initial conditions (OUTER TUBE):

$$\left. \begin{array}{l} \text{At } x = 75 \text{ mm.}, \sigma_x = 8 \text{ N/mm}^2 \\ \text{Substitute in Eqn (3)} \end{array} \right\} \sigma_x = \frac{B'}{x^2} - A' \text{ --- (3)}$$
$$8 = \frac{B'}{75^2} - A' \text{ --- (7)}$$

$$\text{At } x = 100 \text{ mm } \sigma_x = 0 \quad 0 = \frac{B'}{100} - A' \text{ --- (8)}$$

Subtracting Eqn (8) from (7) and simplifying

$$B' = 102,857$$

$$A' = 10.28$$

$$\left. \begin{array}{l} \text{Hoop stress at } x = 75 \text{ mm.} \\ \text{Substituting } A' \text{ and } B' \text{ in (4)} \end{array} \right\} \sigma_H = \frac{102,857}{75^2} + 10.28$$
$$= \underline{28.56 \text{ N/mm}^2} \text{ (Tensile)}$$

$$\left. \begin{array}{l} \text{Hoop stress at } x = 100 \text{ mm.} \\ \text{Substituting in (4)} \end{array} \right\} \sigma_H = \frac{102,857}{100^2} + 10.28$$
$$= \underline{20.57 \text{ N/mm}^2} \text{ (Tensile)}$$

FINAL STRESSES - (After Fluid is admitted):

Lames Equations for compound cylinder

$$\text{Radial Stress } \sigma_r = \frac{B''}{x^2} - A''$$

$$\text{Hoop Stress } \sigma_H = \frac{B''}{x^2} + A''$$

Radial Stress $\sigma_r = 65 \text{ N/mm}^2$ at $x = 60 \text{ mm}$. Substituting above

$$65 = \frac{B''}{60^2} - A'' \quad \text{--- (9)}$$

Again $\sigma_r = 0$ at $x = 100$.

$$0 = \frac{B''}{100^2} - A'' \quad \text{--- (10)}$$

Subtracting (10) from (9) and simplifying

$$B'' = 365625$$

$$A'' = 36.56$$

Hoop Stress - Substituting B'' & A'' in Hoop Stress Eqn

$$\text{At } x = 60 \text{ mm} \Rightarrow \sigma_H = \frac{365625}{60^2} + 36.56$$

$$= \underline{138.12 \text{ N/mm}^2}$$

$$\text{At } x = 100 \text{ mm} \Rightarrow \sigma_H = \frac{365625}{100^2} + 36.56$$

$$= \underline{73.12 \text{ N/mm}^2}$$

A compound cylinder is formed by shrinking one tube on to another, the final dimensions being, internal dia 120 mm, external dia 240 mm, dia at junction 180 mm. If after shrinking on, the radial pressure at the common surface is 8 N/mm^2 . Calculate the initial hoop stresses across the sections of the inner and outer tubes.

If a fluid under a pressure of 60 N/mm^2 is admitted inside the compound cylinder, calculate the final stresses set up in the sections of the pipes.

SOLUTION:

$$d = 120 \text{ mm} \Rightarrow r = 60 \text{ mm}; \quad D = 240 \text{ mm} \Rightarrow R = 120 \text{ mm}.$$

$$d' = 180 \text{ mm} \Rightarrow r' = 90 \text{ mm}; \quad \sigma' = 8 \text{ N/mm}^2$$

$$\sigma_r = 60 \text{ N/mm}^2$$

Lame's Equation for inner tube: (Initial Condition Before fluid is admitted)

→ Radial pressure $\sigma_r = \frac{B}{x^2} - A$

at $x = 60 \text{ mm}; \sigma_r = 0 \Rightarrow 0 = \frac{B}{60^2} - A$ — (1)

at $x = 90 \text{ mm}; \sigma_r = 8 \text{ N/mm}^2 \Rightarrow 8 = \frac{B}{90^2} - A$ — (2)

Subtracting (2) from (1) $B = -51840$
 $A = -14.4$

→ Hoop stress $\sigma_H = \frac{B}{x^2} + A$

Substituting 'B' and 'A' in the above equation

at $x = 60 \text{ mm}; \sigma_H = \frac{-51840}{60^2} - 14.4 = \underline{\underline{28.80 \text{ N/mm}^2}}$

at $x = 90 \text{ mm}; \sigma_H = \frac{-51840}{90^2} - 14.4 = \underline{\underline{20.80 \text{ N/mm}^2}}$

(16)

Outer Tube (Initial Conditions - Before admitting fluid)
Lame's Equation - Radial pressure σ_r

$$\sigma_r = \frac{B'}{x^2} - A'$$

At $x = 90 \text{ mm}$; $\sigma_r = 8 \text{ N/mm}^2 \Rightarrow 8 = \frac{B'}{90^2} - A'$ — (3)

At $x = 120 \text{ mm}$; $\sigma_r = 0 \Rightarrow 0 = \frac{B'}{120^2} - A'$ — (4)

Subtracting (4) from (3)

$$B' = 148114.28$$

$$A' = 10.29$$

→ Hoop stress $\sigma_H = \frac{B'}{x^2} + A'$

At $x = 90 \text{ mm}$; $\sigma_H = \frac{148114.28}{90^2} + 10.29 = 28.58 \text{ N/mm}^2$

At $x = 120 \text{ mm}$; $\sigma_H = \frac{148114.28}{120^2} + 10.29 = 20.57 \text{ N/mm}^2$

FINAL STRESSES (After fluid is admitted):

Lame's Eqn. - Radial stress $\sigma_r = \frac{B''}{x^2} - A''$

At $x = 60 \text{ mm}$; $\sigma_r = 60 \text{ N/mm}^2 \Rightarrow 60 = \frac{B''}{60^2} - A''$ — (5)

At $x = 120 \text{ mm}$; $\sigma_r = 0 \Rightarrow 0 = \frac{B''}{120^2} - A''$ — (6)

Subtracting (6) from (5)

$$B'' = 288000$$

$$A'' = 20$$

→ Hoop stress $\sigma_H = \frac{B''}{x^2} + A''$

At $x = 60 \text{ mm}$; $\sigma_H = \frac{288000}{60^2} + 20 = 100 \text{ N/mm}^2$

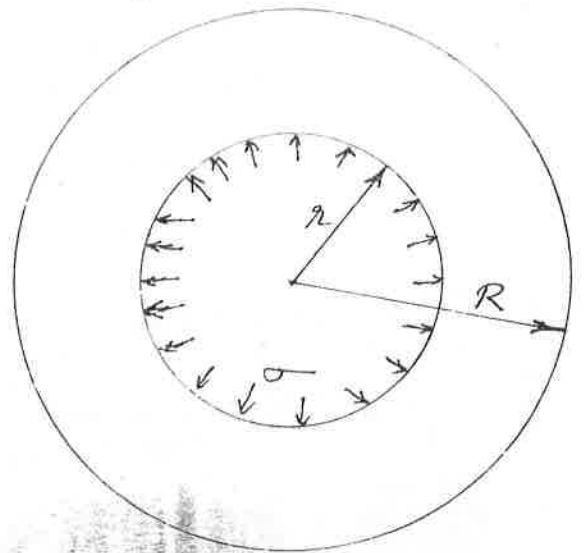
At $x = 120 \text{ mm}$; $\sigma_H = \frac{288000}{120^2} + 20 = 40 \text{ N/mm}^2$

THICK SPHERICAL SHELLS

Consider a thick spherical shell of internal radius ' r ' and external radius ' R ', subjected to internal fluid pressure σ .

→ Radial pressure ' σ_r ' at any radial distance is given by the equation

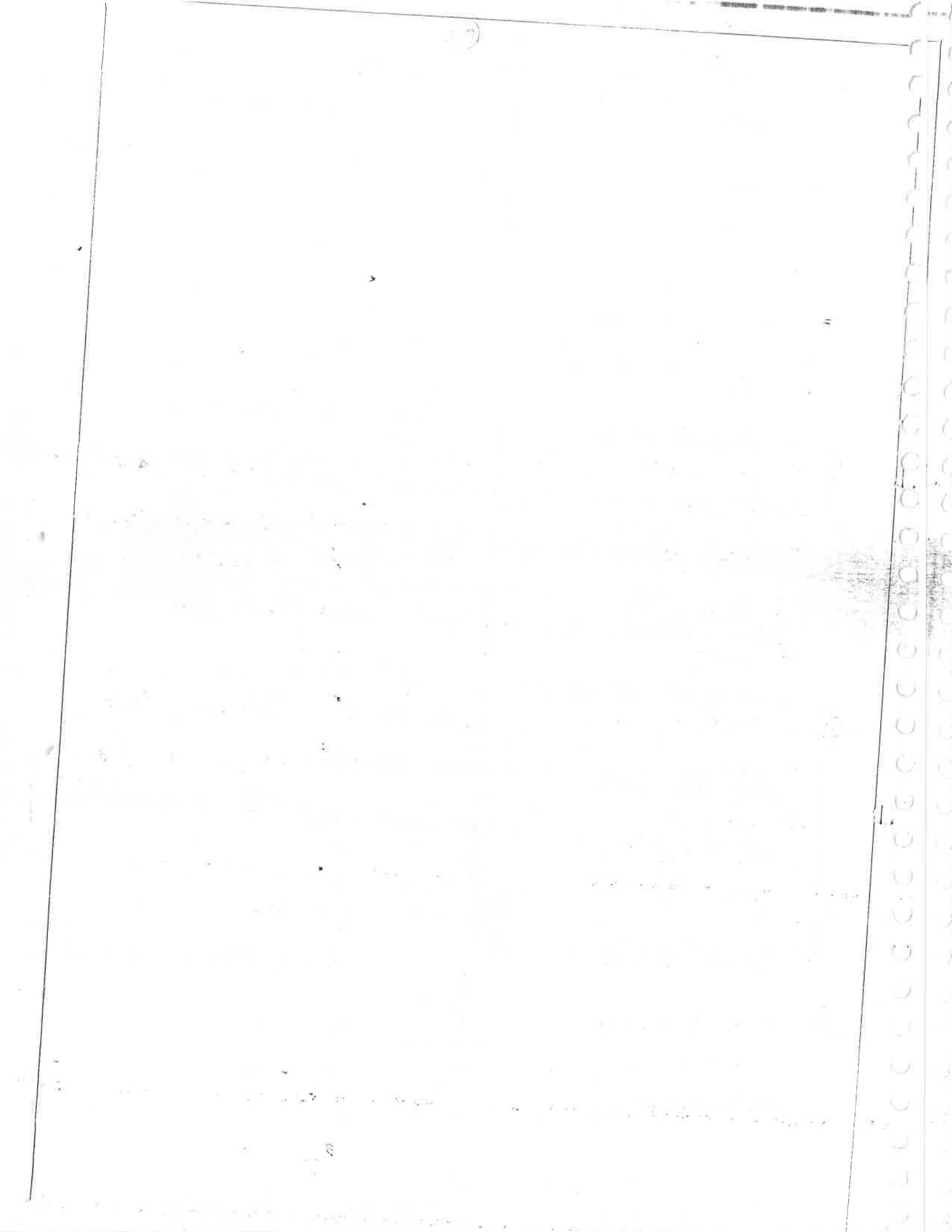
$$\sigma_r = \frac{2B}{r^3} - A$$



→ Circumferential stress or Hoop stress: σ_H

$$\sigma_H = \frac{B}{r^3} + A$$

Applying the boundary conditions, values of ' B ' and ' A ' may be obtained.



SHORT QUESTIONS

① In a thick cylinder radial pressure, as per Lamé's theory, is given by the expression

- (a) $\frac{B}{x^2} + A$ (b) $\frac{2B}{x^2} - A$ (c) $\frac{B}{x^2} - A$ (d) $\frac{2B}{x^2} - A$

② Distribution of Hoop stress across the thickness of a thick cylinder is given by the expression

- (a) $\frac{2B}{x^2} + A$ (b) $\frac{B}{x^2} + A$ (c) $\frac{2B}{x^2} - A$ (d) $\frac{B}{x^2} - A$

③ A thick cylinder is subjected to an internal pressure of 20 N/mm^2 , the radial pressure and at inner radius 'r' and at external radius 'R' are equal to

- (a) zero and 20 N/mm^2 (b) 20 N/mm^2 and zero
(c) Both 20 N/mm^2 (d) Both zero.

④ A thick cylinder is subjected to internal fluid pressure ' σ ', maximum Hoop stress (Tension) occurs at

- (a) Surface of cylinder (b) Inner surface
(c) mid point of thickness (d) none

⑤ Stresses in thick cylinders are determined using

- (a) Rankin's Equations (b) Lamé's Equations (c) Poisson's equations.

⑥ In a compound cylinder

- (a) Thickness $< \frac{1}{10}$ dia (b) Thickness $> \frac{1}{10}$ dia
(c) One cylinder is shrunk on the other

⑦ A compound cylinder is subjected to an internal fluid pressure of 65 N/mm^2 , the internal and external radii of the cylinder are 100 mm and 160 mm . The radial pressure on internal and external radii are ----- respectively.

- (a) 130 N/mm^2 and 65 N/mm^2 (b) 65 N/mm^2 and Zero
(c) 65 N/mm^2 and 130 N/mm^2 (d) Zero and 65 N/mm^2 .

⑧ One cylinder is shrunk over the other at a shrink pressure of 10 N/mm^2 . Radial pressure on the inner and outer surface of external cylinder are

- (a) Zero and 10 N/mm^2 (b) 10 N/mm^2 and Zero
(c) 5 N/mm^2 on both surfaces (d) none of the above.

⑨ In thick spherical shell radial pressure is given by the expression

- (a) $\frac{B}{x^3} - A$ (b) $\frac{2B}{x^3} - A$ (c) $\frac{2B}{x^2} - A$ (d) $\frac{B}{x^2} - A$.

⑩ Hoop stress in thick spherical shells is given by the expression

- (a) $\frac{B}{x^3} - A$ (b) $\frac{B}{x^3} + A$ (c) $\frac{2B}{x^3} + A$ (d) $\frac{2B}{x^3} - A$.