The Following are the innovative tools used by the Faculty in Teaching and Learning Process

1. **Group Assignment/ Project**

Instructors can structure a Group Assignment so that each member of the group must submit the assignment or the Group Assignment can be structured so that any member of the group can submit for the entire group.

2. **Models & Charts to give better grasping**

Instructors can use different charts to explain the algorithms and various models in technical oriented concepts that can create an awareness regarding their academics

3. **Role Play**

Faculty members are using role playing and scenario analysis based teaching as another Innovative method. Instructors can supplement their teaching methods with role playing in any context where it seems relevant. Even rehearsals of personal situations through role playing with a trusted friend can provide beneficial learning opportunities.

4. **Guest Lectures**

Our Department encourages guest lecturers to motivate the Students and also improve the thinking knowledge related to the current trends in technology.

5.E-class Room

Faculty is using E-class room for interactive session. LCD Projector is used for demonstration, video (NPTEL), audio of classes. The faculty members are using multimedia elements such as Tabs and LCD projectors in the classroom. It will help the faculty members to represent the content in a more meaningful way using different media elements.

Due to current ongoing Pandemic situation across the whole world , Faculty is using different online teaching platforms such as ZOOM, MS TEAMS, GOOGLE MEET etc

Fig 2.11 E-Class Room

6. Quizzes

A quiz can function throughout a course as an informative feedback device allowing both the instructor and the Students to see where they are excelling or need more focus. In order to effectively create Exams and **quizzes**, it is important to establish and understand the learning objectives that are being measured.

7.Soft Skill Class

Understanding the need of one's personality enables an individual to act more genuinely and effectively in a team environment. Students are encouraged to deliver presentations in the class which help them to develop ability to gather information, make decisions and interact with others. Soft skills classes empower Students with confidence, boldness, expressiveness etc. Also the Students' personality is developed overall.

C.	**Methodologies to support weak Students and encourage bright Students**

The Coordinator regularly conduct meetings regarding progress of their mentors and are responsible to identify Students who scored less than 60% Marks in their internals. Under the HOD direction, the coordinator evaluates the progress card of those Students who score below 60% Marks in three or more subject and below 75% attendance are considered as **academically weak Students** and same is also intimated to their parents.

Methodology to support weak Students: The Department has a well-defined process of monitoring, guiding and assisting slow learners (weak Students). Teachers attempt to enhance the performance of weak Students as follows:

- Care is taken by the faculty in monitoring the performance of slow learners, the Students deviations from studies is observed by the respective section class teacher and corrective measures are suggested.
- The faculty also goes a step ahead and has periodic interaction with the parents about the performance of slow learners.
- A blended motivation and responsibility from both parents and faculty will create a positive mindset and will help to overcome the inabilities and hurdles faced by the slow learners.
- Every parent is informed about the Marks and the Attendance.
- Regular counseling and providing moral support to them by counselor. For each counselor around 20 Students are allotted for counseling.
- Additional coaching is given to slow learners through Remedial classes.
- Tutorial classes are conducted by the faculty for those Students who have failed in any subject.
- Students are counseled for regular attendance.
-

Methodology to support bright student:

- The bright Students are identified based on their overall performance and their orientation towards Academics.
- The Department of Mechanical Engineering always has the culture of encouraging bright Students by providing them necessary guidance and moral support.
- Encouraging them to score good percentile in their final Examination by providing special and challenging assignments.
- Encouraging them to participate in seminars/conferences in different institutes.
- Encouraging them to participate in state and national levels quiz and debate competitions.
- Students are encouraged to present their ideas in Workshops, Seminars, and Competitive Exams and also in various events.
- Encouraging them to guide their weak classmates. Teaching others make them more perfect.

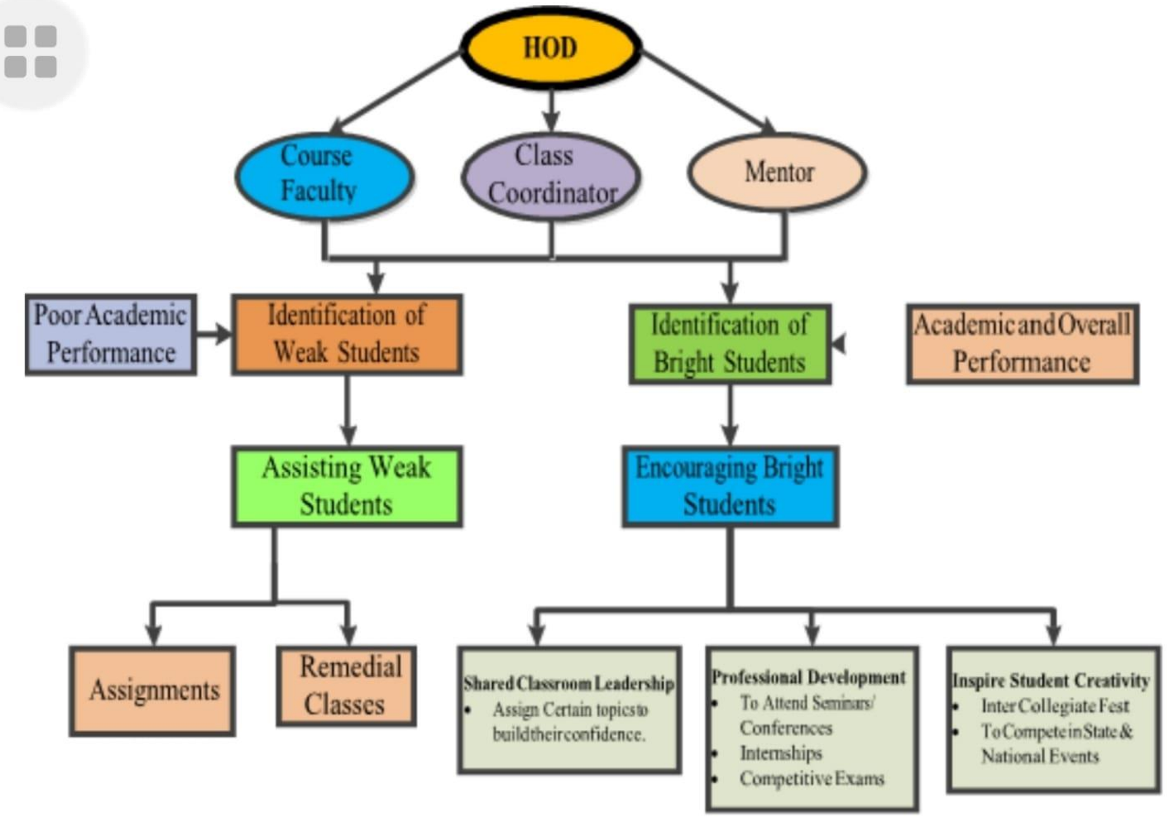


Fig 2.13 Process for Encouraging Bright Students and Assisting Weak Students

Impact analysis:

The following are the positive outcomes observed after adopting the above-mentioned Innovative Teaching Learning Process (TLP):

- Improved attendance of Students for every class.
- Improved Pass Percentage.
- Achieving awards and rewards for their participation in and outside the campus.
- New view points and new project ideas are derived in class.
- Better bondage between Students and faculty.
- Appreciation from the parents.

D.	**Quality of classroom teaching (Observation in a class)**

- Faculty maintains Teaching plan, Tutorial classes list, Teaching notes, Attendance registers, teaching diaries relative to their subject.
- Duration of each Theory Session is 50 minutes, Laboratory session is 3 Hrs.
- The faculty of Department adopts various innovative Teaching & Learning methodologies to create the best learning environment for Students.
- Lectures are delivered to Students as per teaching plan.
- Faculty provides brief summary of last class before the start of new topic.
- Computers are used for teaching purposes and internet facility is available to Students and faculty.
- Faculty members are taking advantage of sources like National Program on Technology Enhanced Learning (NPTEL), internet sources for effective teaching.
- ICTs are used for teaching purposes.

E.	**Conducts of Experiments (Observation in Lab)**

- Course coordinator along with the faculty are involved in preparation of Laboratory manual.

- The Mechanical Engineering Laboratories are conducted in duration of 3 hours with the faculty demonstrating the procedure of the experiment.
- Two faculty members and one instructor are assigned for each lab session. This guides the Students to understand and perform experiment easily.
- The Students perform the experiment and note the output of the program in the observation book.
- The performance of each student in the laboratory during the three-hour laboratory session is evaluated for 10 Marks.
- The executed experiment is documented by the Students in the record book and is evaluated for 5 Marks.
- Each student prepares a lab Record which is assessed by the teacher before commencement of the next practical.
- In each laboratory, the Students are trained to perform experiments on content beyond syllabus for better understanding/performance and to meet the industry requirements.
- The Internal Laboratory Exam is evaluated for 10 Marks.
- The total Internal Assessment is evaluated for **25 Marks** (Day to Day Performance **(10M)** + Record **(5M)** + Internal Exam **(10M)**).

As per the University, Curriculum stipulates a Minimum of 2 laboratory courses or a Maximum of 4 laboratory courses per semester from **I to VII** semester. As per the University guidelines 10–12 experiments are to be conducted. Students carry out more than the required number of experiments, beyond the minimum specified by the University.

F.	**Continuous Assessment in the Laboratory**

Continuous Assessment system is also implemented for assessment of laboratory work. The Assessment is done on the basis of Day to Day Performance, Laboratory Record and Internal Lab Examination.

JNTUH LAB ASSESSMENT

S.No	Day to Day Performance	Record Assessment	Internal Lab Examination	TOTAL
1	10 Marks	5 Marks	10 Marks	**25 Marks**

OU LAB ASSESSMENT

S.No	Day to Day Performance	Record Assessment	Internal Lab Examination	TOTAL
1	10 Marks	10 Marks	10 Marks	**30Marks**

22.2 Student List with Slow Learners on the Basis of CNC TECH MID-1 Exam

S. No.	Roll No.	Student Name	Mid-1 Total (25M)
1	17RT1A0331	MOHAMMED ABDUL WASY	18
2	17RT1A0333	MOHAMMED ABIDULLAH ANSARI	20
3	17RT1A0336	MOHAMMED ASAD AHMED	20
4	17RT1A0343	MOHAMMED IMRAN	20
5	17RT1A0344	MOHAMMED INZEMAMUDDIN	20
6	17RT1A0356	MOHAMMED SHAHBAZ HUSSAIN	20
7	17RT1A0361	MOHAMMED VASIUDDIN	20
8	17RT1A0368	MOAHD ABDUL RAHMAN	20
9	17RT1A0374	MOHD FAISAL HUSSAIN	20
10	17RT1A0377	MOHD IMRAN UDDIN	19
11	17RT1A0383	MOHD PARVEZ	20
12	17RT1A0393	SHAIK ABDUL OBAID	20
13	17RT1A0395	SHAIK ASHRAF ALI	20
14	17RT1A03A0	SHAIK SAMI UR RAHMAN	20
15	17RT1A03A2	SK MOHAMMED NAIF UDDIN	20
16	17RT1A03B5	SYED MOHD LATEEF	20
17	17RT1A03B8	SYED NOOR MOHAMMED	20
18	17RT1A03C1	SYED TALIB AZAM	20
19	17RT1A0394	SHAIK ABDUL WASI	16
20	17RT1A03A6	SYED ESA GIBRAN	19
21	17RT1A03A7	SYED FARDEEN ALI	19
22	18RT5A0304	HABEEB AHMED	20
23	18RT5A0305	IBRAHIM BIN HASAN MOHAMMADI	20

24	18RT5A0308	M A WASEEM	20
25	18RT5A0302	AHMED ABDUL HAQUE	18
26	18RT5A0303	CHEGONDI SIVALINGA RAJU	15
27	18RT5A0306	KAMA NAVEEN	18
28	18RT5A0310	M A MUNAWAR	19
29	18RT5A0312	MOHAMMAD ABDUL RAHMAN	20
30	18RT5A0313	MOHAMMAD FARDEEN FARAZ	20
31	18RT5A0315	MOHAMMED ABDUL KHALEEL	20
32	18RT5A0316	MOHAMMED ABDUL RAHMAN	20
33	18RT5A0317	MOHAMMED IBRAHIM	20
34	18RT5A0319	MOHAMMED SHOAIB KHAN	20
35	18RT5A0326	MOHD SADIQ	20
36	18RT5A0327	MOHD SHOAIB ABBAS	20
37	18RT5A0329	SHAIK AWAIS	20
38	18RT5A0331	SHAIK UMAR SHARIEF	20

23. CO & PO Attainment

Nawab Shah Alam Khan College of
Engineering & Technology

Applied Thermodynamics

Department of
Mechanical Engineering

Course Objectives

- To study the application of thermal science in mechanical engineering, consisting of the fundamental laws and processes for energy conversion
- To understand thermal design aspects of reciprocating machinery-reciprocating compressors and IC Engines.
- To analyze Rankine cycle applied to thermal power plants and its improvements.
- To gain the knowledge on the power plant thermal Devices-Boilers, Condensers, Pumps &Nozzles.

Course Outcomes

1. Expected to be able to quantify the behavior of reciprocating compressors.
2. Expected to be able to explain thermal design and working principles of IC Engines, their supporting systems and Combustion chambers.
3. Expected to be able to quantify the behavior of power plants based on the Rankine cycle, including the effect of enhancements such as superheat, reheat and regeneration.

Course Outcomes

4. Expected to be able to explain the thermal design and working principles of Power plant devices.
5. Expected to be able to explain working principles of Boilers, Condensers, Pumps &Nozzles.

Syllabus

Unit I

Reciprocating Air Compressors:

- Classification and applications.
- Ideal and actual P-V diagrams
- Work input and efficiency relations for single and multi-stage compressors.
- Effect of clearance volume on work input and efficiency.
- Inter cooling and after cooling concepts.

Syllabus

- **Unit II - Internal Combustion Engines**
- **Unit III - I.C. Engine Combustion phenomena**
- **Unit IV - Steam Boilers, Steam Condensers**
- **Unit V - Steam Power Plant Cycles, Steam Nozzles**

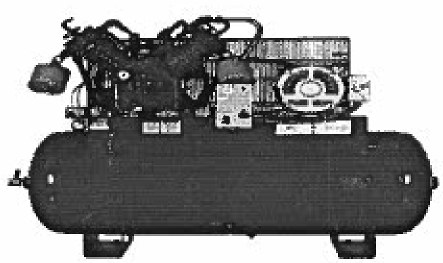
Suggested Reading

- R.K. Rajput, " Thermal Engineering", Laxmi Publications, 9th Edn., 2013
- V. Ganesan, "Internal Combustion Engines", Tata McGraw Hill Publishing, 2007
- P.L. Ballaney, "Thermal Engineering", Khanna Publishers, 19th Edn., 1993.
- Richard Stone, "Introduction to I.C. Engines", Mac Millan, 2nd Edn., 1997

How they are different from pumps?

- Major difference is that compressors handle gases and pumps handle liquids.
- As gases are compressible, the compressor also reduces the volume of a gas.
- Liquids are relatively incompressible, while some can be compressed.

Reciprocating Air Compressors



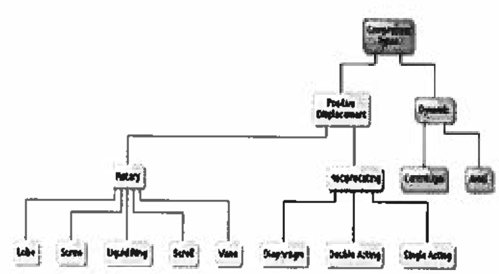
Applications

Compressors have many everyday uses, such as in :

- Air conditioners, (car, home)
- Pneumatic devices
- Home and industrial refrigeration
- Hydraulic compressors for industrial machines
- Air compressors for industrial manufacturing

Classification of Compressor

Compressor classification can be described by following flow chart:



Dynamic Compressors

The dynamic compressor is a continuous flow compressor and is characterized by rotating impeller to add velocity and thus pressure to fluid.

It is widely used in chemical and petroleum refinery industry for specific services.

There are two types of dynamic compressors

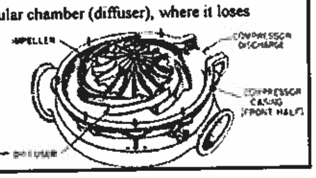
- Centrifugal Compressor
- Axial Flow Compressor

Axial Flow Compressor

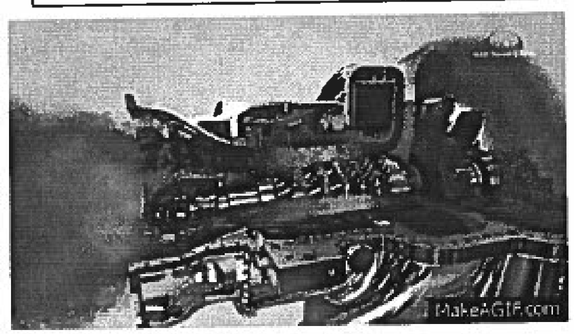
- Working fluid principally flows parallel to the axis of rotation.
- The energy level of air or gas flowing through it is increased by the action of the rotor blades which exert a torque on the fluid
- Have the benefits of high efficiency and large mass flow rate
- Require several rows of airfoils to achieve large pressure rise making them complex and expensive

Centrifugal Compressors

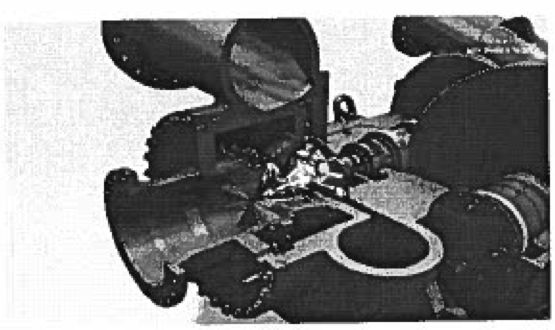
- Achieves compression by applying inertial forces to the gas by means of rotating impellers.
- It is multiple stage , each stage consists of an impeller as the rotating element and the stationary element, i.e. diffuser
- Fluid flow enters the impeller axially and is discharged radially
- The gas next flows through a circular chamber (diffuser), where it loses velocity and increases pressure.



Axial Flow Compressor



Centrifugal Compressors



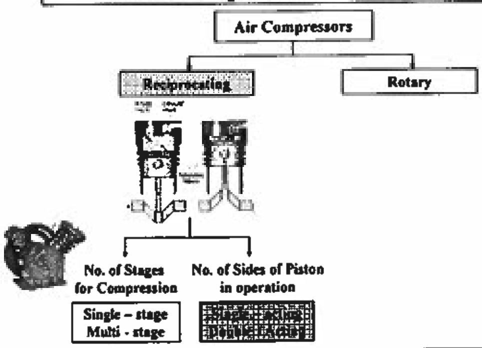
Positive displacement Compressor

Positive displacement compressors work by trapping a fixed amount of air in a chamber and then reducing the volume of the chamber to compress the air.

It can be further classified according to the mechanism used to move air.

- Rotary Compressor
- Reciprocating compressor

Positive displacement Compressor

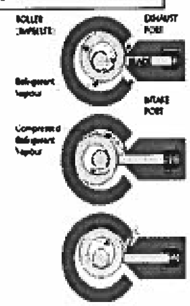


Important Terms

- **Inlet Pressure:** Absolute pressure of air at the inlet of the compressor.
- **Discharge Pressure:** Absolute pressure of air at the outlet of compressor.
- **Compression Ratio (Pressure Ratio):** Ratio of discharge pressure to inlet pressure. Always more than unity.
- **Compressor Capacity:** Volume of air delivered by compressor. Expressed in m^3/min or m^3/s.
- **Free air delivery:** Actual volume of air delivered by a compressor when reduced to the normal temperature and pressure condition. Capacity of compressor is generally given in terms of free air delivery.

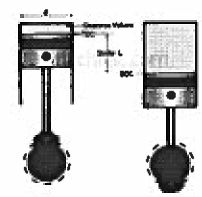
Rotary Compressor

- The gas is compressed by the rotating action of a roller inside a cylinder.
- The roller rotates off-centre around a shaft so that part of the roller is always in contact with the cylinder.
- Volume of the gas chamber is reduced and the gas is compressed.
- Highly efficient as suction and compression of gas occur simultaneously.



Important Terms

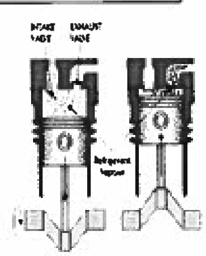
- Swept Volume: [illegible]
- Mean effective pressure: [illegible]



Reciprocating Compressor

It is a positive-displacement compressor that

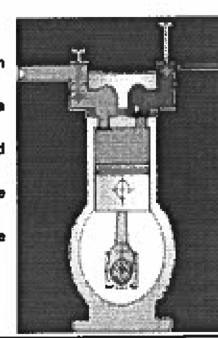
- Uses pistons driven by a crankshaft to deliver gases at high pressure.
- The intake gas enters the cylinder through suction manifold, then flows into the compression chamber
- It gets compressed by a piston driven in a reciprocating motion via a crankshaft,
- Discharged at higher pressure



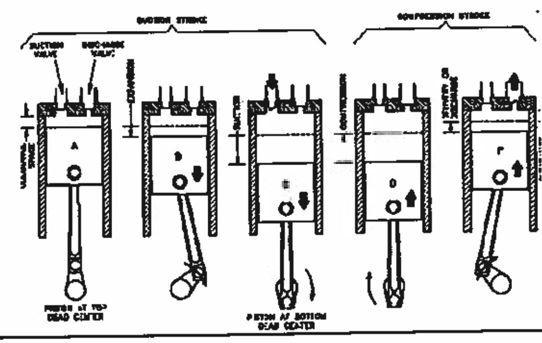
Reciprocating Compressor - Detailed Analysis

Principle of Operation

- Fig. shows single-acting piston movement in the cylinder of a reciprocating compressor.
- The piston is driven by a crank shaft via a connecting rod.
- At the top of the cylinder is a suction valve and a discharge valve.
- The valves are opened because of pressure difference.
- Reciprocating compressors can have multiple stages of compression.



Reciprocating Compressor - Working



Why multistage compressor?

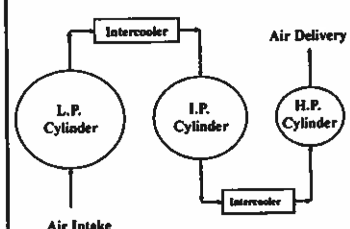
- High temp rise leads into limitation for the maximum achievable pressure rise.
- Discharge temperature shall not exceed 150°C and should not exceed 135°C for hydrogen rich services
- A multistage compressor compresses air to the required pressure in multiple stages.
- Intercoolers are used in between each stage to removes heat and decrease the temperature of gas so that gas could be compressed to higher pressure without much rise in temperature

Factors which decrease Volumetric Efficiency

1. Very high speed.
2. Leakage past the piston
3. Large clearance volume
4. Obstruction at inlet valves
5. Heating of air by contact with hot cylinder walls
6. Inertia effect of air in suction pipe.

Reciprocating Compressor – Multistage

Series arrangement of cylinders, in which the compressed air from earlier cylinder (i.e. *discharge*) becomes the intake air for the next cylinder (i.e. *inlet*).



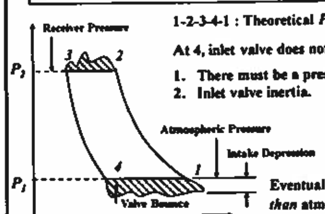
L.P. = Low Pressure

I.P. = Intermediate Pressure

H.P. = High Pressure

Intercooler :
Compressed air is *cooled* between cylinders.

Reciprocating Compressor – Actual *P-V* Diagram



1-2-3-4-1 : Theoretical *P-V* Diagram.

At 4, inlet valve does not open because:

1. **There must be a pressure difference across the valve to open.**
2. **Inlet valve inertia.**

Pr. Drop continues till sufficient level for valve to force its seat.

Some *valve bounce* is set (wavy line).

Eventually, the pressure sets down at a level *lower than* atmospheric pressure. This negative pressure difference is known as *Intake Depression*.

Similar situation appears at 2, i.e. at the start of the delivery.

Pressure rise, followed by valve bounce and then pressure settles at a level *higher than* the delivery pressure level.

Air delivery to a tank / receiver, hence, generally known as *Receiver Pressure*.

Reciprocating Compressor – Performance

Isothermal work done / cycle = Area of $P - V$ Diagram

$= P_1V_1 \log_e(P_2/P_1)$

$= 2.3\ P_1V_1 \log(P_2/P_1)$

Isothermal Power $=$ Isothermal work done / cycle $\times \frac{N}{60\times1000}\ kW$

Indicated work done $=$ Work done during polytropic compression

$$= \frac{n}{n-1}\times P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Indicated Power $=$ Indicated work done $\times \frac{N}{60\times1000}\ kW$

Reciprocating Compressor – Performance

Isothermal/Compressor Efficiency $\eta_c = \frac{\text{Isothermal work done}}{\text{Indicated work done}} = \frac{\text{Isothermal power}}{\text{Indicated power}}$

Overall Isothermal Efficiency $\eta_o = \frac{\text{Isothermal power}}{\text{Shaft power or B.P. of motor}}$

Mechanical Efficiency $\eta_m = \frac{\text{Indicated power}}{\text{Shaft power or B.P. of motor}}$

Isentropic/Adiabatic Efficiency $\eta_i = \frac{\text{Isentropic power}}{\text{Shaft power or B.P. of motor}}$

(for isentropic efficiency replace n with γ in overall isothermal efficiency.)

Volumetric Efficiency $\eta_{v(amb)} = \frac{\text{Free air delivered per stroke}}{\text{Swept volume of piston}}$

$$= \frac{P_1 T_a}{P_a T_1}\left[1 + K - K\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

$$= \frac{P_1 T_a}{P_a T_1}\left[1 + K - K\left(\frac{V_2}{V_1}\right)\right]$$

(When the ambient and suction conditions are same, then $P_a = P_1$ and $T_a = T_1$)

Reciprocating Compressor – Efficiency

How to Increase Isothermal Efficiency ?

A. Spray Injection : Injects water into the compressor cylinder towards the end of compression stroke.
Object is to cool the air.

Demerits : 1. Requires *special gear* for injection.
2. Injected water interferes with the *cylinder lubrication*.
3. Damage to *cylinder walls* and *valves*.
4. Water must be separated before delivery of air.

B. Water Jacketing : Circulating water around the cylinder to help for cooling the air during compression.

Reciprocating Compressor – Efficiency

How to Increase Isothermal Efficiency ?

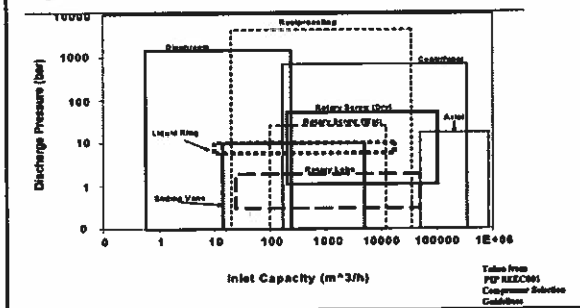
C. Inter – Cooling : For high speed and high Pr. Ratio compressors.
Compressed air from earlier stage is cooled to its original temperature before passing it to the next stage.

D. External Fins : For small capacity compressors, fins on external surfaces are useful.

E. Cylinder Proportions : Short stroke and large bore provides much greater surface for cooling.
Cylinder head surface is far more effective than barrel surface.

How to select a particular type of compressor ?

Graph showing operating regions of various compressors



Taken from PIP REEC001 Compressor Selection Guidelines

Table showing operating conditions of various compressors

Table 1a. Summary of Typical Operating Characteristics of Compressors (US Units)

	Inlet Capacity (cfm)	Maximum Discharge Pressure (psig)	Efficiency (%)	Operating Speed (rpm)	Maximum Power (HP)	Application
Dynamic Compressors						
Centrifugal	100 - 200,000	10,000	70 - 87	1,800 - 50,000	50,000+	Process gas & air
Axial	30,000 - 500,000	200	87 - 90+	1,700 - 10,000	100,000	Mostly air
Positive Displacement Compressors						
Reciprocating (Piston)	10 - 22,000	[illegible]	[illegible]	200 - 900	30,000	Air & process gas
Diaphragm	[illegible]	[illegible]	40 - 70	300 - 500	[illegible]	Corrosive & hazardous process gas
Rotary Screw (Wet)	[illegible] - 7,000	250	45 - 70	1,500 - 3,600	[illegible]	Air, refrigeration & process gas
Rotary Screw (Dry)	120 - 50,000	15 - 700	55 - 70	1,000 - 30,000	[illegible]	Air & dirty process gas
Rotary Lobe	[illegible]	5 - 25	[illegible]	[illegible] - 4,000	[illegible]	Pneumatic conveying, [illegible] gas & vacuum
Sliding Vane	10 - 3,000	150	40 - 70	400 - 1,800	[illegible]	Vacuum service & process gas
Liquid Ring	5 - [illegible]	[illegible]	25 - 50	200 - 3,600	[illegible]	Vacuum service & [illegible] process gas

Taken from PIP REEC001 Compressor Selection Guidelines

Advantages and Disadvantages of Dynamic compressors

	Advantages	Disadvantages
Dynamic Compressors		
Centrifugal	•Wide operating range •High reliability •Low Maintenance	•Instability at reduced flow •Sensitive to gas composition change
Axial	•High Capacity for given size •High efficiency •Heavy duty •Low maintenance	•Low Compression ratios •Limited turndown

Advantages and Disadvantages of Positive displacement compressors

	Advantages	Disadvantages
Positive displacement compressor		
Reciprocating	•Wide pressure ratios •High efficiency	•Heavy foundation required •Flow pulsation •High maintenance
Diaphragm	•Very high pressure •Low flow •No moving seal	•Limited capacity range •Periodic replacement of diaphragm
Screw	•Wide application	•Expensive

Thank You

Nawab Shah Alam Khan College of Engineering and Technology,
New Malakpet, Hyderabad

STEADY HEAT CONDUCTION
Chapter 2 & 3

By
Dr. Noor Alam
Department of Mechanical Engineering, NSACET

SOLUTION OF STEADY ONE-DIMENSIONAL HEAT CONDUCTION PROBLEMS

In this section we will solve a wide range of heat conduction problems in rectangular, cylindrical, and spherical geometries.

We will limit our attention to problems that result in *ordinary differential equations* such as the *steady one-dimensional* heat conduction problems. We will also assume *constant thermal conductivity*.

The solution procedure for solving heat conduction problems can be summarized as

(1) *formulate* the problem by obtaining the applicable differential equation in its simplest form and specifying the boundary conditions,

(2) Obtain the *general solution* of the differential equation, and

(3) apply the *boundary conditions* and determine the arbitrary constants in the general solution.

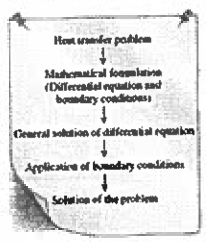


FIGURE 2–39
Basic steps involved in the solution of heat transfer problems.

EXAMPLE 2–10 Heat Conduction in a Plane Wall

Consider a large plane wall of thickness $L = 0.2$ m, thermal conductivity $k = 1.2$ W/m·K, and surface area $A = 15$ m². The two sides of the wall are maintained at constant temperatures of $T_1 = 120°C$ and $T_2 = 50°C$, respectively, as shown in Fig. 2–40. Determine (*a*) the variation of temperature within the wall and the value of temperature at $x = 0.1$ m and (*b*) the rate of heat conduction through the wall under steady conditions.

SOLUTION A plane wall with specified surface temperatures is given. The variation of temperature and the rate of heat transfer are to be determined.

Assumptions 1 Heat conduction is steady. 2 Heat conduction is one-dimensional since the wall is large relative to its thickness and the thermal conditions on both sides are uniform. 3 Thermal conductivity is constant. 4 There is no heat generation.

Properties The thermal conductivity is given to be $k = 1.2$ W/m·K.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x*-direction, the differential equation for this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 120°C$$
$$T(L) = T_2 = 50°C$$

The differential equation is linear and second order, and a quick inspection of it reveals that it has a single term involving derivatives and no terms involving the unknown function T as a factor. Thus, it can be solved by direct integration. Noting that an integration reduces the order of a derivative by one, the general solution of the differential equation above can be obtained by two simple successive integrations, each of which introduces an integration constant.

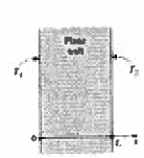


FIGURE 2–40
Schematic for Example 2–10.

Integrating the differential equation once with respect to x yields

$$\frac{dT}{dx} = C_1$$

where C_1 is an arbitrary constant. Notice that the order of the derivative went down by one as a result of integration. As a check, if we take the derivative of this equation, we will obtain the original differential equation. This equation is not the solution yet since it involves a derivative.

Integrating one more time, we obtain

$$T(x) = C_1x + C_2$$

which is the general solution of the differential equation (Fig. 2–41). The general solution in this case resembles the general formula of a straight line whose slope is C_1 and whose value at $x = 0$ is C_2. This is not surprising since the second derivative represents the change in the slope of a function, and a zero second derivative indicates that the slope of the function remains constant. Therefore, *any straight line* is a solution of this differential equation.

The general solution contains two unknown constants C_1 and C_2, and thus we need two equations to determine them uniquely and obtain the specific solution. These equations are obtained by forcing the general solution to satisfy the specified boundary conditions. The application of each condition yields one equation, and thus we need to specify two conditions to determine the constants C_1 and C_2.

When applying a boundary condition to an equation, *all occurrences of the dependent and independent variables and any derivatives are replaced by the specified values*. Thus the only unknowns in the resulting equations are the arbitrary constants.

The first boundary condition can be interpreted as *in the general solution, replace all the x's by zero and T(x) by T_1*. That is (Fig. 2–42),

$$T(0) = C_1 \times 0 + C_2 \rightarrow C_2 = T_1$$

FIGURE 2–41
Obtaining the general solution of a simple second order differential equation by integration.

The second boundary condition can be interpreted as *in the general solution, replace all the x's by L and T(x) by T_2*. That is,

$$T(L) = C_1L + C_2 \rightarrow T_2 = C_1L + T_1 \rightarrow C_1 = \frac{T_2 - T_1}{L}$$

Substituting the C_1 and C_2 expressions into the general solution, we obtain

$$T(x) = \frac{T_2 - T_1}{L}x + T_1 \quad (2\text{-}56)$$

which is the desired solution since it satisfies not only the differential equation but also the two specified boundary conditions. That is, differentiating Eq. 2–56 with respect to x twice will give d^2T/dx^2, which is the given differential equation, and substituting $x = 0$ and $x = L$ into Eq. 2–56 gives $T(0) = T_1$ and $T(L) = T_2$, respectively, which are the specified conditions at the boundaries.

Substituting the given information, the value of the temperature at $x = 0.1$ m is determined to be

$$T(0.1\text{ m}) = \frac{(50 - 120)°C}{0.2\text{ m}}(0.1\text{ m}) + 120°C = 85°C$$

(*b*) The rate of heat conduction anywhere in the wall is determined from Fourier's law to be

$$\dot{Q}_{wall} = -kA\frac{dT}{dx} = -kAC_1 = -kA\frac{T_2 - T_1}{L} = kA\frac{T_1 - T_2}{L} \quad (2\text{-}57)$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (1.2\text{ W/m·K})(15\text{ m}^2)\frac{(120 - 50)°C}{0.2\text{ m}} = 6300\text{ W}$$

Discussion Note that under steady conditions, the rate of heat conduction through a plane wall is constant.

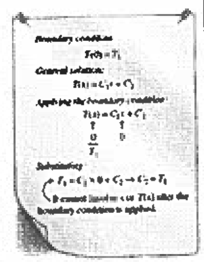


FIGURE 2–42
When applying a boundary condition to the general solution at a specified point, all occurrences of the dependent and independent variables should be replaced by their specified values at that point.

STEADY HEAT CONDUCTION IN PLANE WALLS

Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*.
The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as $T(x)$.

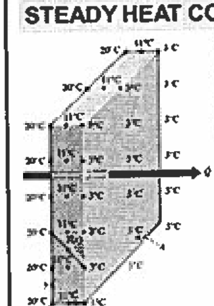


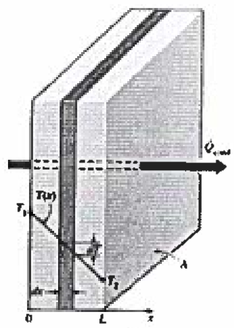
FIGURE 3–1
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

$$\begin{pmatrix}\text{Rate of}\\\text{heat transfer}\\\text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\\text{heat transfer}\\\text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\\text{of the energy}\\\text{of the wall}\end{pmatrix}$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt} \qquad dE_{wall}/dt = 0 \text{ for steady operation}$$

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{cond,\,wall} = -kA\frac{dT}{dx} \quad \text{(W)} \quad \text{Fourier's law of heat conduction}$$



$$\dot{Q}_{cond,\,wall} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{cond,\,wall}\,dx = -\int_{T=T_1}^{T_2} kA\,dT$$

$$\dot{Q}_{cond,\,wall} = kA\frac{T_1 - T_2}{L} \qquad (W)$$

The rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

Once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 by T, and L by x.

Under steady conditions, the temperature distribution in a plane wall is a straight line: dT/dx = const.

EXAMPLE 1.2.
A plane wall of 10 cm thickness and 3 m² area is made of a material whose conductivity is 0.5 W/mK. The temperatures of the wall surfaces are steady at 100°C and 30°C respectively. Find the temperature gradient and heat flow across the wall.

EXAMPLE 1.5.
[illegible]

EXAMPLE 1.6.
[illegible] ... Take convective heat transfer coefficient to be 3000 W/m² deg.

EXAMPLE 1.11.
[illegible]

EXAMPLE 1.13.
A surface at 200°C loses heat both by convection and radiation to the surroundings at 50°C. The convection coefficient is 75 W/m²K and the radiation factor due to geometric location and emissivity is 0.95. If the heat is conducted to the surface through a solid material of thermal conductivity 10 W/mK, determine the temperature gradient at the surface of the solid.

Thermal Resistance Concept

$$\dot{Q}_{cond,\,wall} = kA\frac{T_1 - T_2}{L}$$

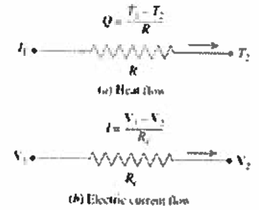
$$\dot{Q}_{cond,\,wall} = \frac{T_1 - T_2}{R_{wall}} \qquad (W)$$

$$R_{wall} = \frac{L}{kA} \qquad (°C/W)$$

Conduction resistance of the wall: *Thermal resistance* of the wall against heat conduction.

Thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

$$I = \frac{V_1 - V_2}{R_e} \qquad R_e = L/\sigma_e A \quad \textit{Electrical resistance}$$



Analogy between thermal and electrical resistance concepts.

rate of heat transfer → electric current
thermal resistance → electrical resistance
temperature difference → voltage difference

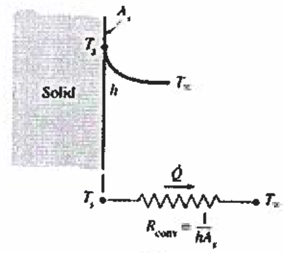
Newton's law of cooling

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad (W)$$

$$R_{conv} = \frac{1}{hA_s} \qquad (°C/W)$$

Convection resistance of the surface: *Thermal resistance* of the surface against heat convection.



Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes zero and $T_s \approx T$.
That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.
This situation is approached in practice at surfaces where boiling and condensation occur.

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad}A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}}$$

$$R_{rad} = \frac{1}{h_{rad}A_s} \qquad (K/W)$$

Radiation resistance of the surface: *Thermal resistance* of the surface against radiation.

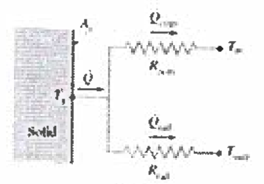
$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \qquad (W/m^2 \cdot K)$$

Radiation heat transfer coefficient

When $T_{surr} \approx T_\infty$

$$h_{combined} = h_{conv} + h_{rad}$$

Combined heat transfer coefficient



Schematic for convection and radiation resistances at a surface.

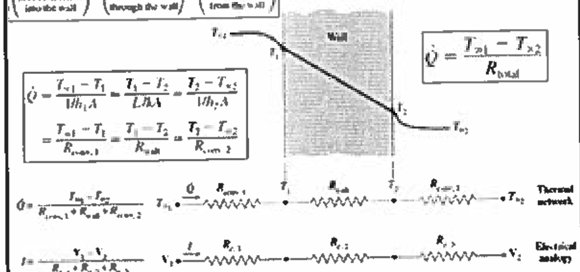
Thermal Resistance Network

$$\left(\text{Rate of heat convection into the wall}\right) = \left(\text{Rate of heat conduction through the wall}\right) = \left(\text{Rate of heat convection from the wall}\right)$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_1 - T_2}{R_{wall}} = \frac{T_2 - T_{\infty 2}}{R_{conv,2}}$$

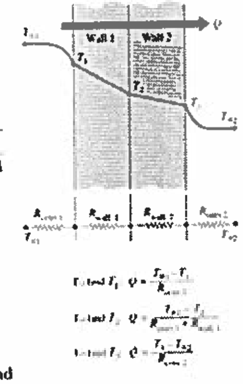


The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°C/W)$$

$$\dot{Q} = \frac{T_i - T_j}{R_{total,\, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{wall,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

Temperature drop

$$\Delta T = \dot{Q}R \qquad (°C)$$

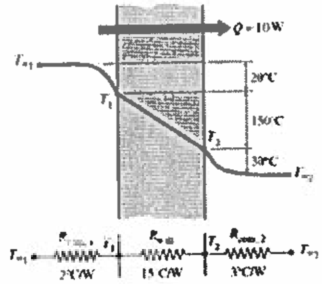
$$\dot{Q} = UA\,\Delta T \qquad (W)$$

$$UA = \frac{1}{R_{total}} \qquad (°C/K)$$

U overall heat transfer coefficient

Once Q is evaluated, the surface temperature T_1 can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$

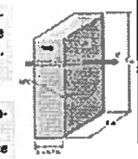


$\Delta T = \dot{Q}R$

The temperature drop across a layer is proportional to its thermal resistance.

EXAMPLE 1 – heat loss through a wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m·K (Fig. 3–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.



SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

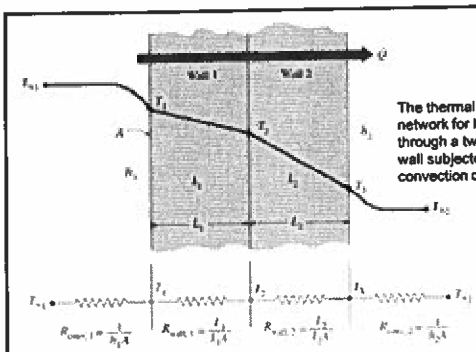
Assumptions 1 Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. 2 Heat transfer is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. 3 Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9$ W/m·K.

Analysis Noting that heat transfer through the wall is by conduction and the area of the wall is $A = 3\text{ m} \times 5\text{ m} = 15\text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9\text{ W/m·°C})(15\text{ m}^2)\frac{(16-2)°C}{0.3\text{ m}} = 630\text{ W}$$

Multilayer Plane Walls



The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$R_{total} = R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{wall}}{R_{wall}}$$

where

$$R_{wall} = \frac{L}{kA} = \frac{0.3\text{ m}}{(0.9\text{ W/m·°C})(15\text{ m}^2)} = 0.02222\text{°C/W}$$

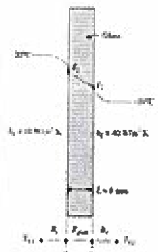
Substituting, we get

$$\dot{Q} = \frac{(16-2)°C}{0.02222\text{°C/W}} = 630\text{ W}$$

Discussion This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples. Also, the units W/m·°C and W/m·K for thermal conductivity are equivalent, and thus interchangeable. This is also the case for °C and K for temperature differences.

EXAMPLE 2 – heat loss through a single-pane window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m·K. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m²·K and $h_2 = 40$ W/m²·K, which includes the effects of radiation.



Assumptions 1 Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. 2 Heat transfer through the wall is one-dimensional since any significant temperature gradients exist in the direction from the indoors to the outdoors. 3 Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m·K.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–12. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_{glass} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00855°\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$R_{total} = R_{conv,1} + R_{glass} + R_{conv,2} = 0.08333 + 0.00855 + 0.02083 = 0.1127°\text{C/W}$$

Then the steady rate of heat transfer through the window becomes

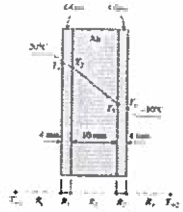
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-10)]°\text{C}}{0.1127°\text{C/W}} = 266 \text{ W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 20°\text{C} - (266 \text{ W})(0.08333°\text{C/W}) = -2.2°\text{C}$$

Discussion Note that the inner surface temperature of the window glass is −2.2°C even though the temperature of the air in the room is maintained at 20°C. Such low surface temperatures are highly undesirable since they cause the formation of fog or even frost on the inner surfaces of the glass when the humidity in the room is high.

EXAMPLE 3 – heat loss through a double-pane window



Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78$ W/m·K) separated by a 10-mm-wide stagnant air space ($k = 0.026$ W/m·K). Determine the steady rate of heat transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m²·K and $h_2 = 40$ W/m²·K, which includes the effects of radiation.

SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem involves two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 3–13. Noting that the area of the window is again $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.08333°\text{C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.00427°\text{C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m}\cdot\text{K})(1.2 \text{ m}^2)} = 0.3205°\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})(1.2 \text{ m}^2)} = 0.02083°\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$R_{total} = R_{conv,1} + R_{glass,1} + R_{air} + R_{glass,2} + R_{conv,2} = 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 = 0.4332°\text{C/W}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-10)]°\text{C}}{0.4332°\text{C/W}} = 69.2 \text{ W}$$

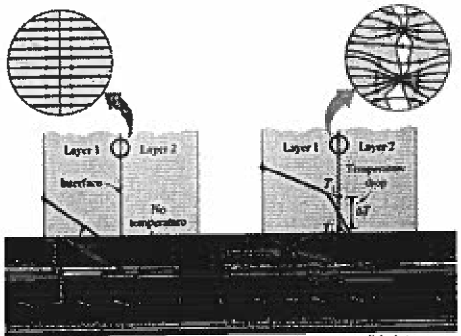
which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 20°\text{C} - (69.2 \text{ W})(0.08333°\text{C/W}) = 14.2°\text{C}$$

which is considerably higher than the −2.2°C obtained in the previous example. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce the heat gain in summer, and thus reduce the air-conditioning costs.

THERMAL CONTACT RESISTANCE



Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

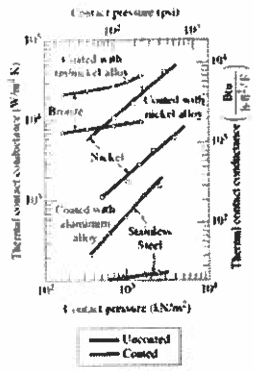
TABLE 3–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, 1969).

Fluid at the interface	Contact conductance, h_c, W/m²·K
Air	3640
Helium	9620
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

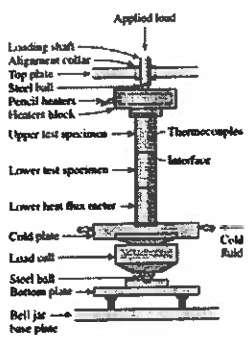
The thermal contact resistance can be minimized by applying

- a *thermal grease* such as silicon oil
- a *better conducting gas* such as helium or hydrogen
- a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum



Effect of metallic coatings on thermal contact conductance

- When two such surfaces are pressed against each other, the peaks form good material contact but the valleys form voids filled with air.
- These numerous *air gaps* of varying sizes act as *insulation* because of the low thermal conductivity of air.
- Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the **thermal contact resistance, R_c.**



A typical experimental setup for the determination of thermal contact resistance

---

TABLE 3–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m²·K
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar Metal Pairs					
Stainless steel–Aluminum		20–30	20	10	2900
				20	3600
Stainless steel–Aluminum		1.0–2.0	20	10	16,400
				20	20,800
Steel Ct-30–Aluminum	Ground	1.4–2.0	20	10	50,000
				15–35	59,000
Steel Ct-30–Aluminum	Milled	4.5–7.2	20	10	4800
				30	8300
Aluminum-Copper	Ground	1.17–1.4	20	5	42,000
				15	56,000
Aluminum-Copper	Milled	4.4–4.5	20	10	12,000
				20–35	22,000

The *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure*.

$$\dot{Q} = \dot{Q}_{contact} + \dot{Q}_{gap}$$

$$\dot{Q} = h_c A \, \Delta T_{interface}$$

h_c thermal contact conductance

$$h_c = \frac{\dot{Q}/A}{\Delta T_{interface}} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C})$$

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{interface}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {}^\circ\text{C/W})$$

The value of thermal contact resistance depends on:

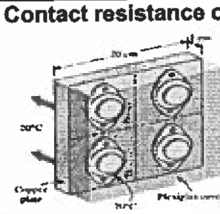
- *surface roughness,*
- *material properties,*
- *temperature* and *pressure* at the interface
- *type of fluid* trapped at the interface.

$$R_{c,\,insulation} = \frac{L}{k} = \frac{0.01 \text{ m}}{0.04 \text{ W/m} \cdot {}^\circ\text{C}} = 0.25 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

$$R_{c,\,copper} = \frac{L}{k} = \frac{0.01 \text{ m}}{386 \text{ W/m} \cdot {}^\circ\text{C}} = 0.000026 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

Thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be disregarded for poor heat conductors such as insulations.

EXAMPLE 4 – Contact resistance of Transistors



Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm × 20-cm square copper plate (k = 386 W/m·K) by screws that exert an average pressure of 6 MPa (Fig. 3–18). The base area of each transistor is 8 cm², and each transistor is placed at the center of a 10-cm × 10-cm quarter section of the plate. The interface roughness is estimated to be about 1.5 μm. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be 25 W/m²·K. If the case temperature of the transistor is not to exceed 70°C, determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.

SOLUTION Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C, the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. 3 All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. 4 Thermal conductivities are constant.

Properties The thermal conductivity of copper is given to be $k = 386$ W/m·K. The contact conductance is obtained from Table 3-2 to be $h_c = 42{,}000$ W/m² K, which corresponds to copper-aluminum interface for the case of 1.17–1.4 μm roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis The contact area between the case and the plate is given to be 8 cm², and the plate area for each transistor is 100 cm². The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{interface} = \frac{1}{h_c A_c} = \frac{1}{(42{,}000\ \text{W/m}^2\cdot\text{K})(8 \times 10^{-4}\ \text{m}^2)} = 0.030°\text{C/W}$$

$$R_{plate} = \frac{L}{kA} = \frac{0.01\ \text{m}}{(386\ \text{W/m}\cdot\text{K})(0.01\ \text{m}^2)} = 0.0026°\text{C/W}$$

$$R_{conv} = \frac{1}{h_o A} = \frac{1}{(25\ \text{W/m}^2\cdot\text{K})(0.01\ \text{m}^2)} = 4.0°\text{C/W}$$

The total thermal resistance is then

$$R_{total} = R_{interface} + R_{plate} + R_{ambient} = 0.030 + 0.0026 + 4.0 = 4.0326°\text{C/W}$$

8/26/2021 31

The total thermal resistance is then

$$R_{total} = R_{interface} + R_{plate} + R_{ambient} = 0.030 + 0.0026 + 4.0 = 4.0326°\text{C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{(70 - 20)°\text{C}}{4.0326°\text{C/W}} = 12.4\ \text{W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C.

The temperature jump at the interface is determined from

$$\Delta T_{interface} = \dot{Q}R_{interface} = (12.4\ \text{W})(0.030°\text{C/W}) = 0.37°\text{C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we lower the operating temperature of the transistor in this case by less than 0.4°C.

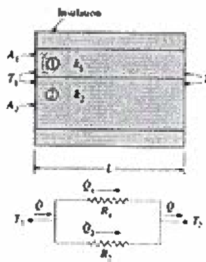
8/26/2021 Department of Mechanical Engineering, NSACET 32

GENERALIZED THERMAL RESISTANCE NETWORKS

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$



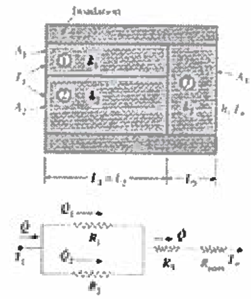
Thermal resistance network for two parallel layers.

8/26/2021 Department of Mechanical Engineering, NSACET 33

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \qquad R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

$$R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{conv} = \frac{1}{hA_3}$$



Two assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are

(1) any plane wall normal to the x-axis is *isothermal* (i.e., to assume the temperature to vary in the x-direction only)

(2) any plane parallel to the x-axis is *adiabatic* (i.e., to assume heat transfer to occur in the x-direction only)

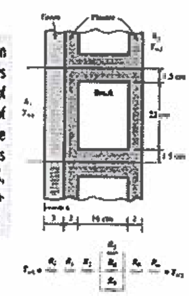
Do they give the same result?

Thermal resistance network for combined series-parallel arrangement.

8/26/2021 Department of Mechanical Engineering, NSACET 34

EXAMPLE 5 – Heat Loss Through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks (k = 0.72 W/m·K) separated by 3-cm-thick plaster layers (k = 0.22 W/m·K). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k = 0.026 W/m·K) on the inner side of the wall, as shown in Fig. 3-21. The indoor and the outdoor temperatures are 20°C and −10°C, respectively, and the convection heat transfer coefficients on the inner and the outer sides are h_1 = 10 W/m²·K and h_2 = 25 W/m²·K, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.



8/26/2021 Department of Mechanical Engineering, NSACET 35

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of change with time. 2 Heat transfer can be approximated as being one-dimensional since it is predominantly in the x-direction. 3 Thermal conductivities are constant. 4 Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be k = 0.72 W/m·K for bricks, k = 0.22 W/m·K for plaster layers, and k = 0.026 W/m·K for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the x-direction to be *isothermal*, the thermal resistance network for the representative section of the wall becomes as shown in Fig. 3-21. The individual resistances are evaluated as:

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 0.40°\text{C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.03\ \text{m}}{(0.026\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)} = 4.62°\text{C/W}$$

$$R_2 = R_6 = R_{plaster,side} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.25 \times 1\ \text{m}^2)}$$

$$= 0.36°\text{C/W}$$

$$R_3 = R_5 = R_{plaster,center} = \frac{L}{kA} = \frac{0.16\ \text{m}}{(0.22\ \text{W/m}\cdot\text{K})(0.015 \times 1\ \text{m}^2)}$$

$$= 48.48°\text{C/W}$$

8/26/2021 Department of Mechanical Engineering, NSACET 36

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.16\ m}{(0.72\ W/m\cdot K)(0.22 \times 1\ m^2)} = 1.01°C/W$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\ W/m^2\cdot K)(0.25 \times 1\ m^2)} = 0.16°C/W$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03\ W/°C$$

which gives

$$R_{mid} = 0.97°C/W$$

Now all the resistances are in series, and the total resistance is

$$R_{total} = R_i + R_1 + R_2 + R_{mid} + R_6 + R_o = 0.40 + 4.62 + 0.36 + 0.97 + 0.36 + 0.16 = 6.87°C/W$$

8/26/2021 37

Department of Mechanical Engineering, NSACET

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-10)]°C}{6.87°C/W} = 4.37\ W \quad \text{(per 0.25 m}^2\text{ surface area)}$$

or 4.37/0.25 = 17.5 W per m² area. The total area of the wall is $A = 3\ m \times 5\ m = 15\ m^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{total} = (17.5\ W/m^2)(15\ m^2) = 263\ W$$

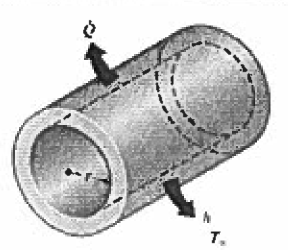
Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion In the above solution, we assumed the temperature at any cross section of the wall normal to the x-direction to be *isothermal*. We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the x-direction to be *adiabatic*. The thermal resistance network in this case will be as shown in Fig. 3-22. By following the approach outlined above, the total thermal resistance in this case is determined to be $R_{total} = 6.97°C/W$, which is very close to the value 6.85°C/W obtained before. Thus either approach gives roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.

8/26/2021 38

Department of Mechanical Engineering, NSACET

HEAT CONDUCTION IN CYLINDERS AND SPHERES



Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

Heat transfer through the pipe can be modeled as *steady* and *one-dimensional*.

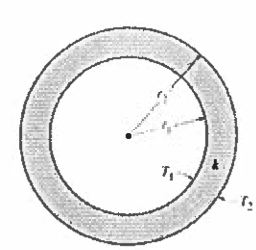
The temperature of the pipe depends on one direction only (the radial r-direction) and can be expressed as $T = T(r)$.

The temperature is independent of the azimuthal angle or the axial distance.

This situation is approximated in practice in long cylindrical pipes and spherical containers.

8/26/2021 39

Department of Mechanical Engineering, NSACET



A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr} \quad (W)$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{cond,cyl}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

$$A = 2\pi rL$$

$$\dot{Q}_{cond,cyl} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \quad (W)$$

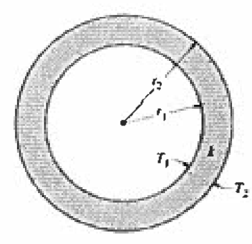
$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}} \quad (W)$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

Conduction resistance of the cylinder layer

8/26/2021 40

Department of Mechanical Engineering, NSACET



A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

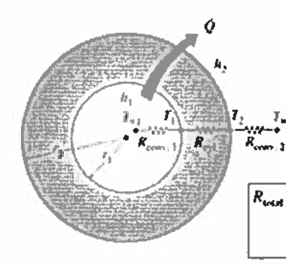
$$\dot{Q}_{cond,sph} = \frac{T_1 - T_2}{R_{sph}}$$

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

Conduction resistance of the spherical layer

8/26/2021 41

Department of Mechanical Engineering, NSACET



$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$

The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

for a *cylindrical* layer

$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2}$$

for a *spherical* layer

$$R_{total} = R_{conv,1} + R_{sph} + R_{conv,2} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

8/26/2021 42

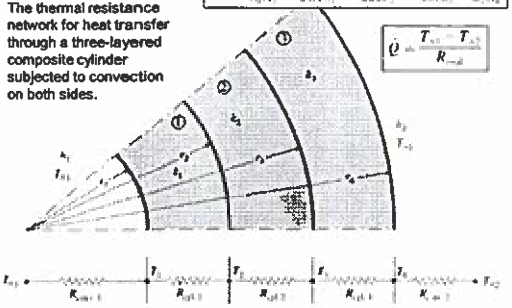
Department of Mechanical Engineering, NSACET

Multilayered Cylinders and Spheres

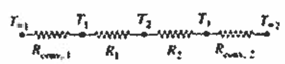
$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2} = \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$



8/26/2021 | 43 | Department of Mechanical Engineering, NSACET



The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_1} = \frac{T_1 - T_3}{R_1 + R_2} = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_{\infty 2}}{R_2 + R_{conv,2}} = \dots$$

Once heat transfer rate Q has been calculated, the interface temperature T_2 can be determined from any of the following two relations:

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{cyl,1}} = \frac{T_{\infty 1} - T_2}{\frac{1}{h_1(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi L k_1}}$$

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{conv,2}} = \frac{T_2 - T_{\infty 2}}{\frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_o(2\pi r_4 L)}}$$

8/26/2021 | 44 | Department of Mechanical Engineering, NSACET

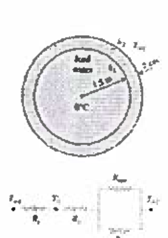
EXAMPLE 6 – Heat Transfer to a Spherical Container

A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel (k = 15 W/m·K) is used to store iced water at $T_{\infty 1}$ = 0°C. The tank is located in a room whose temperature is $T_{\infty 2}$ = 22°C. The walls of the room are also at 22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are h_1 = 80 W/m²·K and h_2 = 10 W/m²·K, respectively. Determine (a) the rate of heat transfer to the iced water in the tank and (b) the amount of ice at 0°C that melts during a 24-h period.

SOLUTION A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions 1 Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. 3 Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be k = 15 W/m·K. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg. The outer surface of the tank is black and thus its emissivity is ε = 1.



8/26/2021 | 45 | Department of Mechanical Engineering, NSACET

Analysis (a) The thermal resistance network for this problem is given in Fig. 3-28. Noting that the inner diameter of the tank is D_1 = 3 m and the outer diameter is D_2 = 3.04 m, the inner and the outer surface areas of the tank are

$$A_1 = \pi D_1^2 = \pi(3\text{ m})^2 = 28.3\text{ m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04\text{ m})^2 = 29.0\text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking T_2 = 5°C = 278 K, the radiation heat transfer coefficient is determined to be

$$h_{rad} = (1)(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(295\text{ K})^2 + (278\text{ K})^2][(295 + 278)\text{ K}]$$
$$= 5.34\text{ W/m}^2\cdot\text{K} = 5.34\text{ W/m}^2\cdot°\text{C}$$

8/26/2021 | 46 | Department of Mechanical Engineering, NSACET

Then the individual thermal resistances become

$$R_i = R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{(80\text{ W/m}^2\cdot\text{K})(28.3\text{ m}^2)} = 0.000442°\text{C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50)\text{ m}}{4\pi(15\text{ W/m}\cdot\text{K})(1.52\text{ m})(1.50\text{ m})}$$
$$= 0.000047°\text{C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{(10\text{ W/m}^2\cdot\text{K})(29.0\text{ m}^2)} = 0.00345°\text{C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A_2} = \frac{1}{(5.34\text{ W/m}^2\cdot\text{K})(29.0\text{ m}^2)} = 0.00646°\text{C/W}$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{equiv}} = \frac{1}{R_o} + \frac{1}{R_{rad}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7\text{ W/°C}$$

which gives

$$R_{equiv} = 0.00225°\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$R_{total} = R_i + R_1 + R_{equiv} = 0.000442 + 0.000047 + 0.00225 = 0.00274°\text{C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(22 - 0)°\text{C}}{0.00274°\text{C/W}} = 8029\text{ W} \quad (\text{or } \dot{Q} = 8.029\text{ kJ/s})$$

8/26/2021 | 47 | Department of Mechanical Engineering, NSACET

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equiv}} \longrightarrow T_2 = T_{\infty 2} - \dot{Q}R_{equiv}$$
$$= 22°\text{C} - (8029\text{ W})(0.00225°\text{C/W}) = 4°\text{C}$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.

(b) The total amount of heat transfer during a 24-h period is

$$Q = \dot{Q}\,\Delta t = (8.029\text{ kJ/s})(24 \times 3600\text{ s}) = 693,700\text{ kJ}$$

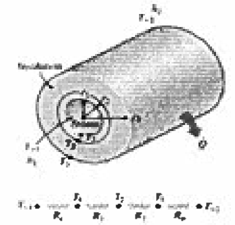
Noting that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the amount of ice that will melt during a 24-h period is

$$m_{ice} = \frac{Q}{h_{if}} = \frac{693,700\text{ kJ}}{333.7\text{ kJ/kg}} = 2079\text{ kg}$$

Therefore, about 2 metric tons of ice will melt in the tank every day.

8/26/2021 | 48 | Department of Mechanical Engineering, NSACET

EXAMPLE 7 – Heat Loss through an Insulated Steam Pipe



Steam at $T_{\infty 1} = 320°C$ flows in a cast iron pipe ($k = 80$ W/m·K) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with $k = 0.05$ W/m·K. Heat is lost to the surroundings at $T_{\infty 2} = 5°C$ by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m²·K. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m²·K, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. 3 Thermal conductivities are constant. 4 The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m·K for cast iron and $k = 0.05$ W/m·K for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{(60 \text{ W/m}^2\cdot\text{K})(0.157 \text{ m}^2)} = 0.106°\text{C/W}$$
$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 0.0002°\text{C/W}$$
$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 2.35°\text{C/W}$$
$$R_o = R_{conv,2} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2\cdot\text{K})(0.361 \text{ m}^2)} = 0.154°\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61°\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(320 - 5)°\text{C}}{2.61°\text{C/W}} = 121 \text{ W} \quad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 3-17 to be

$$\Delta T_{pipe} = \dot{Q}R_{pipe} = (121 \text{ W})(0.0002°\text{C/W}) = 0.02°\text{C}$$
$$\Delta T_{insulation} = \dot{Q}R_{insulation} = (121 \text{ W})(2.35°\text{C/W}) = 284°\text{C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

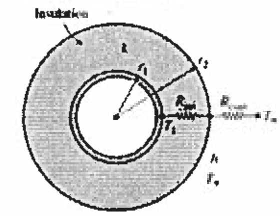
Discussion Note that the thermal resistance of the pipe is too small relative to the other resistances and can be neglected without causing any significant error. Also note that the temperature drop across the pipe is practically zero, and thus the pipe can be assumed to be isothermal. The resistance to heat flow in insulated pipes is primarily due to insulation.

CRITICAL RADIUS OF INSULATION

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

In a a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

The heat transfer from the pipe may increase or decrease, depending on which effect dominates.



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$

The critical radius of insulation for a cylindrical body:

$$r_{cr,\,cylinder} = \frac{k}{h} \quad \text{(m)}$$

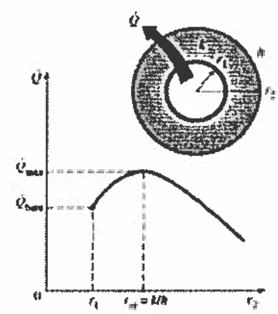
The critical radius of insulation for a spherical shell:

$$r_{cr,\,sphere} = \frac{2k}{h}$$

The largest value of the critical radius we are likely to encounter is

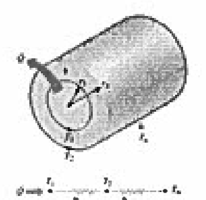
$$r_{cr,\,max} = \frac{k_{max,\,insulation}}{h_{min}} \approx \frac{0.05 \text{ W/m}\cdot°\text{C}}{5 \text{ W/m}^2\cdot°\text{C}} = 0.01 \text{ m} = 1 \text{ cm}$$

We can insulate hot-water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.



The variation of heat transfer rate with the outer radius of the insulation r_2 when $r_1 < r_{cr}$.

EXAMPLE 8 – Heat Loss from an Insulated Electric Wire



A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m·K. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°C$ with a heat transfer coefficient of $h = 12$ W/m²·K, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. 3 Thermal conductivities are constant. 4 The thermal contact resistance at the interface is negligible. 5 Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be k = 0.15 W/m·K.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8\ \text{V})(10\ \text{A}) = 80\ \text{W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\ \text{m})(5\ \text{m}) = 0.110\ \text{m}^2$$

$$R_{conv} = \frac{1}{hA_2} = \frac{1}{(12\ \text{W/m}^2\cdot\text{K})(0.110\ \text{m}^2)} = 0.76°\text{C/W}$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\ \text{W/m}\cdot\text{K})(5\ \text{m})} = 0.18°\text{C/W}$$

and therefore

$$R_{total} = R_{plastic} + R_{conv} = 0.76 + 0.18 = 0.94°\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{total}$$

$$= 30°\text{C} + (80\ \text{W})(0.94°\text{C/W}) = 105°\text{C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{cr} = \frac{k}{h} = \frac{0.15\ \text{W/m}\cdot\text{K}}{12\ \text{W/m}^2\cdot\text{K}} = 0.0125\ \text{m} = 12.5\ \text{mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will enhance heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Discussion It can be shown by repeating the calculations above for a 4-mm-thick plastic cover that the interface temperature drops to 90.6°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 83°C when the outer radius of the plastic cover equals the critical radius.

Nawab Shah Alam Khan College of Engineering and Technology, New Malakpet, Hyderabad

Introduction of Heat Transfer; Conduction, Convection and Radiation

By
Dr. Noor Alam
Department of Mechanical Engineering, NSACET

Heat Transfer

- Heat always moves from a warmer place to a cooler place.
- Hot objects in a cooler room will cool to room temperature.
- Cold objects in a warmer room will heat up to room temperature.

Question

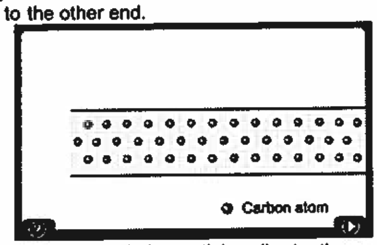
- If a cup of coffee and a red popsickle were left on the table in this room what would happen to them? Why?
- The cup of coffee will cool until it reaches room temperature. The popsickle will melt and then the liquid will warm to room temperature.

Heat Transfer Methods

- Heat transfers in three ways:
  - Conduction
  - Convection
  - Radiation

Conduction

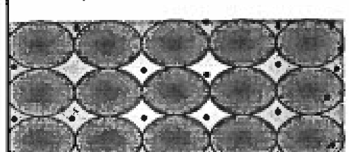
When you heat a metal strip at one end, the heat travels to the other end.



As you heat the metal, the particles vibrate, these vibrations make the adjacent particles vibrate, and so on and so on, the vibrations are passed along the metal and so is the heat. We call this? Conduction

Metals are different

The outer electrons of metal atoms drift, and are free to move.

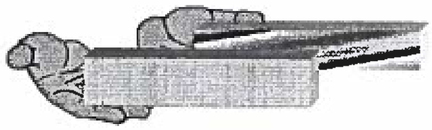


When the metal is heated, this 'sea of electrons' gain kinetic energy and transfer it throughout the metal.

Insulators, such as wood and plastic, do not have this 'sea of electrons' which is why they do not conduct heat as well as metals.

Why does metal feel colder than wood, if they are both at the same temperature?

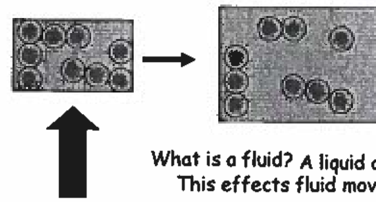
Metal is a conductor, wood is an insulator. Metal conducts the heat away from your hands. Wood does not conduct the heat away from your hands as well as the metal, so the wood feels warmer than the metal.



Convection

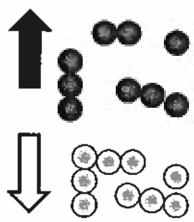
What happens to the particles in a liquid or a gas when you heat them?

The particles spread out and become less dense.



What is a fluid? A liquid or gas.
This effects fluid movement.

Fluid movement

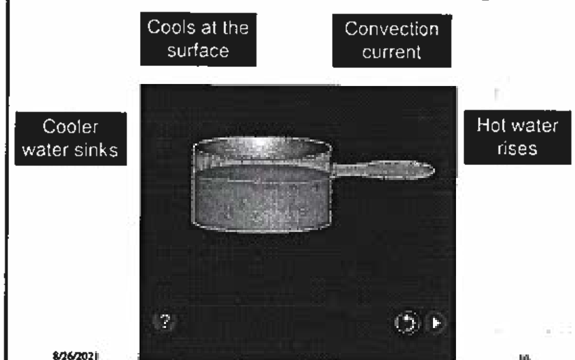


Cooler, more dense, fluids sink through warmer, less dense fluids.

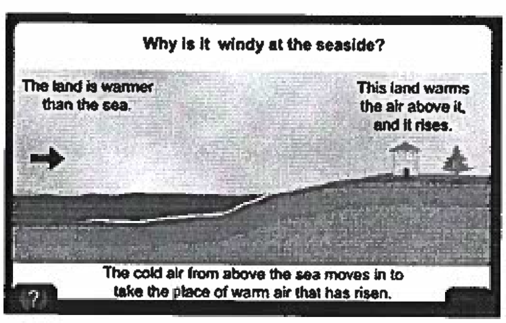
In effect, warmer liquids and gases rise up.

Cooler liquids and gases sink.

Water movement



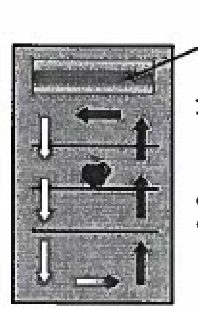
Why is it windy at the seaside?



Cold air sinks

Where is the freezer compartment put in a fridge?

It is put at the top, because cool air sinks, so it cools the food on the way down.

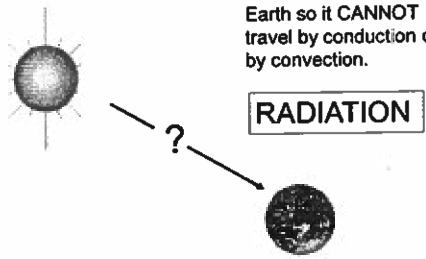


It is warmer at the bottom, so this warmer air rises and a convection current is set up.

The third method of heat transfer

How does heat energy get from the Sun to the Earth?

There are no particles between the Sun and the Earth so it CANNOT travel by conduction or by convection.



Radiation

Radiation travels in straight lines

True/~~False~~

Radiation can travel through a vacuum

True/~~False~~

Radiation requires particles to travel

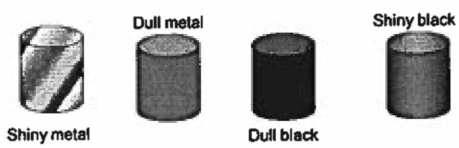
~~True~~/False

Radiation travels at the speed of light

True/~~False~~

Emission experiment

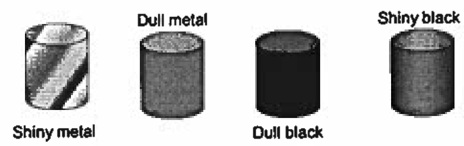
Four containers were filled with warm water. Which container would have the warmest water after ten minutes?



The shiny metal container would be the warmest after ten minutes because its shiny surface reflects heat radiation back into the container so less is lost. The dull black container would be the coolest because it is the best at emitting heat radiation.

Absorption experiment

Four containers were placed equidistant from a heater. Which container would have the warmest water after ten minutes?



The dull black container would be the warmest after ten minutes because its surface absorbs heat radiation the best. The shiny metal container would be the coolest because it is the poorest at absorbing heat radiation.

Convection questions

Why does hot air rise and cold air sink?

Cool air is more dense than warm air, so the cool air 'falls through' the warm air.

Why are boilers placed beneath hot water tanks in people's homes?

Hot water rises.

So when the boiler heats the water, and the hot water rises, the water tank is filled with hot water.

Radiation questions

Why are houses painted white in hot countries?

White reflects heat radiation and keeps the house cooler.

Why are shiny foil blankets wrapped around marathon runners at the end of a race?

The shiny metal reflects the heat radiation from the runner back in, this stops the runner getting cold.

1. Which of the following is not a method of heat transfer?

A. Radiation
(B.) Insulation
C. Conduction
D. Convection

2. In which of the following are the particles closest together?

(A.) Solid
B. Liquid
C. Gas
D. Fluid

3. How does heat energy reach the Earth from the Sun?

(A.) Radiation
B. Conduction
C. Convection
D. Insulation

4. Which is the best surface for reflecting heat radiation?

(A.) Shiny white
B. Dull white
C. Shiny black
D. Dull black

5. Which is the best surface for absorbing heat radiation?

A. Shiny white
B. Dull white
C. Shiny black
(D.) Dull black